Introduction to Astrophysics
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Proton-Proton Chain

In the first reaction $p + p \to d + e^+ + \nu$ in the proton-proton chain we can take $\mu = m_p/2$ and $Z_1 = Z_2 = 1$, and then for $T = 10^7$ K which is the temperature at the center of the Sun, the Coulomb barrier suppreses the reaction by a factor $\exp(-15.7) = 1.5 \times 10^{-7}$.

The reaction $p + p \to d + e^+ + \nu$ is just the first step in a chain. The Coulomb barrier suppression of the second step, $^1H +^2 H \to^3 H_e + \gamma$, is only slightly more severe than the one in the first reaction. This is because the charges of the nuclei are the same, and their reduced mass is larger only by a factor $4/3$. Taking $\mu = 2m_p/3$, $Z_1 = Z_2 = 1$, and $T = 10^7$ K in Eq. (158) gives $B_T \approx \exp(-4/3 \times 15.7) = 8 \times 10^{-10}$. Apart from Coulomb suppression, since step I involves a weak interaction it is suppressed by an additional factor of order 10^{-22} and since step II involves an electromagnetic interaction it is suppressed by an additional factor of order $1/137$, so the ratio of the rate per proton of step I and the rate per deuteron of step II is expected to be of order

$$\frac{\text{rate}/p \quad \text{of} \quad p + p \to d + e^+ + \nu}{\text{rate}/d \quad \text{of} \quad p + d \to^3 H_e + \gamma} \approx \frac{10^{-22} \times 1.5 \times 10^{-7}}{(1/137) \times (8 \times 10^{-10})} \simeq 3 \times 10^{-18}$$

The actual ratio is about one order of magnitude bigger (10^{-17}). Reaction III has a larger Coulomb barrier, with $Z_1 Z_2 = 4$. All three reactions release substantial amounts of energy. The question is then: which ones have to be calculated in order to find ϵ? which one is the relevant Coulomb barrier?

To answer this question we need to look at the time-dependence of these reactions. The abundances of the intermediate participants in these reactions rapidly evolve to stable values, which change little over times during which a very large number of reactions take place in the star's core. Consequently, in order that the abundance of deuterons should not change, the rates per volume of reactions I and II, in which deuterons are respectively created and destroyed, should be the same, and in order that the abundance of ${}^3H_e$ nuclei should not change, the rate per volume of reaction II should be twice that of reaction III, in which two ${}^3H_e$ nuclei are destroyed:

$$\Gamma \equiv \Gamma(I) = \Gamma(II) = 2\Gamma(III), \tag{160}$$

where the Γs denote the rates per volume of various reactions. This relationship express a tendency to equilibrium. If the rate per volume of reaction II were less than that of reaction I the abundance of ${}^2H$ nuclei would rise until these rates were equal, and just as many ${}^2H$ nuclei were being destroyed as created. According to the above estimate of the ratio of the rate per proton of reaction I and the rate per deuteron of reaction II, we therefore expect the number density of deuterons to be smaller than the number density of protons by a factor of order 3×10^{-18}. Though $\Gamma(I)$, $\Gamma(II)$, and $2\Gamma(III)$ must all be equal, their calculation differs in one important respect. The rate of reaction I does not depend on the abundance of the intermediate nuclei ${}^2H$ and ${}^3H_e$, and in particular is not suppressed by their low abundance, so it can be calculated without knowing anything about the other reactions. Thus it is the Coulomb barrier in reaction I that governs the rate at which hydrogen is converted to helium and energy is produced, and then its dependence with temperature.
In particular, in accordance with the general relationship (159), for the proton–proton cycle the exponent ν in the temperature dependence of ϵ is one-third of the value 15.7 that we previously calculated for the exponent in the barrier penetration factor for reaction I, so $\nu \simeq 5$ at $T \approx 10A^7$ K. Fortunately ν has only a mild dependence on temperature, going as $T^{-1/3}$, so this estimate of ν is a fair approximation for a wide range of temperatures. Although we only need to calculate the rate Γ of reaction I, all of reactions I, II, and III release energy, an amount of energy E_I, E_{II}, and E_{III} per reaction, so the rate $\epsilon\rho$ of total energy production per volume is not just $E_I\Gamma$, but

$$\epsilon\rho = \left(E_I + E_{II} + \frac{1}{2}E_{III}\right)\Gamma = 13.1\mathrm{MeV} \times \Gamma \tag{161}$$

The crucial first step in the proton–proton chain is a collision of two protons. Its rate, and consequently the rate per volume $\epsilon\rho$ of energy generation due to the proton–proton chain, is proportional to ρ^2. So, if the proton–proton chain dominates nuclear energy generation, we have $\lambda = 1$ as well as $\nu = 5$. The reactions (149) in the proton-proton chain dominate the energy production, but there are alternative final routes to this chain, one of which is of historical importance. In one alternative, instead of a pair of ^3H_e nuclei combining in reaction III, individual ^3H_e nuclei undergo the reaction

$$\mathrm{III}': \qquad {}^3\mathrm{H_e} +^4 \mathrm{H_e} \rightarrow^7 \mathrm{B_e} + \gamma \tag{162}$$

followed either by

$$\begin{aligned} \mathrm{IV}: &\quad {}^7\mathrm{B_e} + \mathrm{e}^- \rightarrow^7 \mathrm{L_i} + \nu_\mathrm{e} \\ \mathrm{V}: &\quad {}^7\mathrm{L_i} +^1 \mathrm{H} \rightarrow^4 \mathrm{H_e} +^4 \mathrm{H_e} \end{aligned} \tag{163}$$

or instead

$$\begin{aligned} \mathrm{IV}': &\quad {}^7\mathrm{B_e} +^1 \mathrm{H} \rightarrow^8 \mathrm{B} + \gamma \\ \mathrm{V}': &\quad {}^8\mathrm{B} \rightarrow^8 \mathrm{B} + \mathrm{e}^+ + \nu_\mathrm{e} \\ \mathrm{VI}': &\quad {}^8\mathrm{B_e} \rightarrow^4 \mathrm{H_e} +^4 \mathrm{H_e} \end{aligned} \tag{164}$$

The probability of a ^3H_e nucleus undergoing the reaction III' rather than III is small, so these alternatives have little effect on the energy generation rate ϵ and its density and temperature dependence, but the high energy of the neutrino from the ^8B beta decay in reaction V' extending up to over 10 MeV, offered an early opportunity of observing neutrinos from the Sun.
The reaction $^{37}Cl + \nu_e \rightarrow^{37} Ar + e^-$ that was used to search for solar neutrinos in the experiments of Davis et al. (R. Davis, D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. 20, 1205 1968) on solar neutrinos is sensitive only to these high-energy neutrinos, not to the much lower-energy neutrinos emitted in the other reactions of the proton–proton chain.

The high Coulomb barriers in reactions III' and IV' make the flux of high-energy neutrinos extremely sensitive to the temperature profile in the Sun. Detailed calculations by John Bahcall (1934-2005) showed that the high-energy neutrinos should be observable in Davis' experiments, but decades of searching did not find them. Finally solar neutrinos were detected using the reaction $^{71}\mathrm{G_a} + \nu_\mathrm{e} \rightarrow ^{71}\mathrm{G_e} + \mathrm{e}^-$, but the observed rate was substantially less than predicted by Bahcall. Either Bahcall's calculations were inaccurate, or something was happening to neutrinos on the way to the Earth.

Neutrinos

The neutrino was postulated first by Wolfgang Pauli in 1930 to explain how beta decay could conserve energy, momentum, and angular momentum (spin). In contrast to Niels Bohr, who proposed a statistical version of the conservation laws to explain the observed continuous energy spectra in beta decay, Pauli hypothesized an undetected particle that he called a "neutron", using the same -on ending employed for naming both the proton and the electron. He considered that the new particle was emitted from the nucleus together with the electron or beta particle in the process of beta decay and had a mass similar to the electron.
James Chadwick discovered a much more massive neutral nuclear particle in 1932 and named it a neutron also, leaving two kinds of particles with the same name. The word "neutrino" entered the scientific vocabulary through Enrico Fermi, who used it during a conference in Paris in July 1932 and at the Solvay Conference in October 1933, where Pauli also employed it. The name (the Italian equivalent of "little neutral one") was jokingly coined by Edoardo Amaldi during a conversation with Fermi at the Institute of Physics of via Panisperna in Rome, in order to distinguish this light neutral particle from Chadwick's heavy neutron.

In Fermi's theory of beta decay, Chadwick's large neutral particle could decay to a proton, electron, and the smaller neutral particle (now called an electron antineutrino):

$$n^0 \rightarrow p^+ e^- + \overline{\nu}_e \tag{165}$$

Fermi's paper, written in 1934, unified Pauli's neutrino with Paul Dirac's positron and Werner Heisenberg's neutron-proton model and gave a solid theoretical basis for future experimental work. The journal Nature rejected Fermi's paper, saying that the theory was "too remote from reality". He submitted the paper to an Italian journal, which accepted it, but the general lack of interest in his theory at that early date caused him to switch to experimental physics.

By 1934, there was experimental evidence against Bohr's idea that energy conservation is invalid for beta decay. At the Solvay conference of that year, measurements of the energy spectra of beta particles (electrons) were reported, showing that there is a strict limit on the energy of electrons from each type of beta decay. Such a limit is not expected if the conservation of energy is invalid, in which case any amount of energy would be statistically available in at least a few decays. The natural explanation of the beta decay spectrum as first measured in 1934 was that only a limited (and conserved) amount of energy was available, and a new particle was sometimes taking a varying fraction of this limited energy, leaving the rest for the beta particle. Pauli made use of the occasion to publicly emphasize that the still undetected neutrino must be an actual particle. The first evidence of the reality of neutrinos came in 1938 via simultaneous cloud chamber measurements of the electron and the recoil of the nucleus.

Direct detection

In 1942, Wang Ganchang first proposed the use of beta capture to experimentally detect neutrinos. In the July 20, 1956 issue of Science, Clyde Cowan, Frederick Reines, Francis B. “Kiko” Harrison, Herald W. Kruse, and Austin D. McGuire published confirmation that they had detected the neutrino, a result that was rewarded almost forty years later with the 1995 Nobel Prize (Reines and Perl for the discovery for the tau lepton).
In this experiment, now known as the Cowan–Reines neutrino experiment, antineutrinos created in a nuclear reactor by beta decay reacted with protons to produce neutrons and positrons:

$$\overline{\nu}_e + p^+ \rightarrow n^0 + e^+ \tag{166}$$

The positron quickly finds an electron, and they annihilate each other. The two resulting gamma rays (γ) are detectable. The neutron can be detected by its capture on an appropriate nucleus, releasing a gamma ray. The coincidence of both events -positron annihilation and neutron capture - gives a unique signature of an antineutrino interaction.
In February 1965, the first neutrino found in nature was identified by a group which included Jacques Pierre Friederich Sellschop. The experiment was performed in a specially prepared chamber at a depth of 3 km in the East Rand (“ERPM”) gold mine near Boksburg, South Africa. A plaque in the main building commemorates the discovery. The experiments also implemented a primitive neutrino astronomy and looked at issues of neutrino physics and weak interactions.

Neutrino flavor

The antineutrino discovered by Cowan and Reines is the antiparticle of the electron neutrino. In 1962, Lederman, Schwartz, and Steinberger showed that more than one type of neutrino exists by first detecting interactions of the muon neutrino (already hypothesized with the name *neutretto*),which earned them the 1988 Nobel Prize in Physics. When the third type of lepton, the tau, was discovered in 1975 at the Stanford Linear Accelerator Center, it was also expected to have an associated neutrino (the tau neutrino). The first evidence for this third neutrino type came from the observation of missing energy and momentum in tau decays analogous to the beta decay leading to the discovery of the electron neutrino. The first detection of tau neutrino interactions was announced in 2000 by the DONUT collaboration at Fermilab; its existence had already been inferred by both theoretical consistency and experimental data from the Large Electron–Positron Collider.

Solar neutrino problem

In the 1960s, the now-famous Homestake experiment made the first measurement of the flux of electron neutrinos arriving from the core of the Sun and found a value that was between one third and one half the number predicted by the Standard Solar Model. This discrepancy, which became known as the solar neutrino problem, remained unresolved for some thirty years, while possible problems with both the experiment and the solar model were investigated, but none could be found. Eventually, it was realized that both were actually correct and that the discrepancy between them was due to neutrinos being more complex than was previously assumed. It was postulated that the three neutrinos had nonzero and slightly different masses, and could therefore oscillate into undetectable flavors on their flight to the Earth.

This hypothesis was investigated by a new series of experiments, thereby opening a new major field of research that still continues. Eventual confirmation of the phenomenon of neutrino oscillation led to two Nobel prizes, to R. Davis, who conceived and led the Homestake experiment, and to A.B. McDonald, who led the SNO experiment, which could detect all of the neutrino flavors and found no deficit.

Oscillation

A practical method for investigating neutrino oscillations was first suggested by Bruno Pontecorvo in 1957 using an analogy with kaon oscillations; over the subsequent 10 years, he developed the mathematical formalism and the modern formulation of vacuum oscillations. In 1985 Stanislav Mikheyev and Alexei Smirnov (expanding on 1978 work by Lincoln Wolfenstein) noted that flavor oscillations can be modified when neutrinos propagate through matter. This so-called Mikheyev–Smirnov–Wolfenstein effect (MSW effect) is important to understand because many neutrinos emitted by fusion in the Sun pass through the dense matter in the solar core (where essentially all solar fusion takes place) on their way to detectors on Earth.

Starting in 1998, experiments began to show that solar and atmospheric neutrinos change flavors. This resolved the solar neutrino problem: the electron neutrinos produced in the Sun had partly changed into other flavors which the experiments could not detect.

Although individual experiments, such as the set of solar neutrino experiments, are consistent with non-oscillatory mechanisms of neutrino flavor conversion, taken altogether, neutrino experiments imply the existence of neutrino oscillations. Especially relevant in this context are the reactor experiment KamLAND and the accelerator experiments such as MINOS. The KamLAND experiment has indeed identified oscillations as the neutrino flavor conversion mechanism involved in the solar electron neutrinos.

Similarly MINOS confirms the oscillation of atmospheric neutrinos and gives a better determination of the mass squared splitting. Takaaki Kajita of Japan, and Arthur B. McDonald of Canada, received the 2015 Nobel Prize for Physics for their landmark finding, theoretical and experimental, that neutrinos can change flavors.

Cosmic neutrinos

As well as specific sources, a general background level of neutrinos is expected to pervade the universe, theorized to occur due to two main sources.

1) Cosmic neutrino background (Big Bang originated)

Around 1 second after the Big Bang, neutrinos decoupled, giving rise to a background level of neutrinos known as the cosmic neutrino background (CNB).

2) Diffuse supernova neutrino background (Supernova originated)

R. Davis and M. Koshiba were jointly awarded the 2002 Nobel Prize in Physics. Both conducted pioneering work on solar neutrino detection, and Koshiba's work also resulted in the first real-time observation of neutrinos from the SN 1987A supernova in the nearby Large Magellanic Cloud. These efforts marked the beginning of neutrino astronomy.

SN 1987A represents the only verified detection of neutrinos from a supernova. However, many stars have gone supernova in the universe, leaving a theorized diffuse supernova neutrino background.

It was speculated that neutrinos must have mass. See colored box for a historical recount of neutrino physics.

The issue was settled by experiments at the Sudbury Neutrino Observatory. By monitoring a large tank of heavy water, experimenters could detect high-energy ^8B neutrinos not only in the reaction $\nu_e + d \to p + p + e^-$, which is sensitive only to electron-type neutrinos, but also in the neutral current process $\nu + d \to p + n + \nu$ which is equally sensitive to neutrinos of all types, electron, muon, and tauon. It turned out that the total flux of neutrinos of all types agreed with Bahcall's calculations, providing a decisive vote in favor of neutrino oscillations. Since then the existence of neutrino oscillations has been confirmed and neutrino masses and mixing angles measured in numerous terrestrial experiments.

CNO Cycle

The CNO cycle is more complex. We also assume that the abundances of the intermediate CNO nuclei settle down to constant values. The constancy of the abundance of 13^N requires that reactions i and ii have the same rate per volume; the constancy of the abundance of 13^C requires that reactions ii and iii have the same rate per volume; and so on, so that all these rates per volume are equal:

$$\Gamma(i) = \Gamma(ii) = \Gamma(iii) = \Gamma(iv) = \Gamma(v) = \Gamma(vi) \equiv \Gamma \tag{167}$$

This rate determines the ratios of the abundances of elements to be found. Each of the rates here is proportional to the number density n of the CNO nucleus in the initial state of the reaction

$$\Gamma(i) = n(^{12}C)R(i), \qquad \Gamma(ii) = n(^{13}N)R(ii), \quad \text{etc.}, \tag{168}$$

with the rate factors R dependent only of the density of hydrogen. For each reaction, R is the rate at which the CNO nucleus in the initial state undergoes that reaction. For example, $R(i)$ is the rate at which any individual ^{12}C nucleus undergoes the reaction $^1H +^{12}C \to^{13} N + \gamma$. Then (167) gives

$$\frac{n(^{13}N)}{n(^{12}C)} = \frac{R(i)}{R(ii)}, \qquad \frac{n(^{13}C)}{n(^{12}C)} = \frac{R(i)}{R(iii)}, \quad etc. \tag{169}$$

We would tempted to find the overall density of the CNO nuclei doing

$$n(CNO) \equiv n(^{12}C) + n(^{13}N) + n(^{13}C) + n(^{14}N) + n(^{15}O) + n(^{15}N). \tag{170}$$

But this does not change in the reactions i through vi, and it's determined by the abundances in the cloud where the star is formed (which is itself determined by the interstellar medium). What can be done is to express the common rate Γ in terms of $n(CNO)$. For this we can use the previous equation and (167) with (168)

$$\begin{aligned}\frac{n(CNO)}{\Gamma} &= \frac{n(^{12}C)}{\Gamma(i)} + \frac{n(^{13}N)}{\Gamma(ii)} + \frac{n(^{13}C)}{\Gamma(iii)} + \frac{n(^{14}N)}{\Gamma(iv)} + \frac{n(^{15}O)}{\Gamma(v)} + \frac{n(^{15}N)}{\Gamma(vi)} \\ &= \frac{1}{R(\mathrm{i})} + \frac{1}{R(\mathrm{ii})} + \frac{1}{R(\mathrm{iii})} + \frac{1}{R(\mathrm{iv})} + \frac{1}{R(\mathrm{v})} + \frac{1}{R(\mathrm{vi})}\end{aligned}$$

which shows that the common rate is

$$\Gamma = n(CNO) \Big/ \left(\frac{1}{R(\mathrm{i})} + \frac{1}{R(\mathrm{ii})} + \frac{1}{R(\mathrm{iii})} + \frac{1}{R(\mathrm{iv})} + \frac{1}{R(\mathrm{v})} + \frac{1}{R(\mathrm{vi})}\right) \tag{171}$$

Note: the harmonic mean is the reciprocal of the arithmetic mean of the reciprocals of a given set of observations. Example: the harmonic mean of 1, 3, 5, and 5 is :

$$\left(\frac{1^{-1} + 3^{-1} + 5^{-1} + 5^{-1}}{4}\right)^{-1} = 2.307 \tag{172}$$

while the average is $14/4 = 3.5$

The common rate of the reactions equals the harmonic mean of what the individual rates would be if the density of the CNO nucleus in each initial state equaled the total density n(CNO). The rate per volume $\epsilon\rho$ of energy generation in the CNO cycle is Γ times the sum of the energies in Eq. (150):

$$\epsilon\rho = \Gamma \times 25.03 \ \ \mathrm{MeV}. \tag{173}$$

Because of the absence of a Coulomb barrier in the beta decays ii and v, these reactions have relatively rapid rates R per CNO nucleus, with mean lives 1/R of 7 minutes and 82 seconds, respectively, while 1/R for all the other reactions in the CNO cycle is at least 10^5 years. Thus the terms 1/R(ii) and 1/R(v) can be neglected in the denominator in Eq. (172). Also, for the same reason, the number density of the CNO nucleus in the initial states of the beta decay reactions is much smaller than the number densities of the other CNO nuclei, and can be neglected in n(CNO). Thus Eq. (172) for the rate Γ of the various reactions in the CNO channel is dominated by the two-body reactions i, iii, iv, and vi. As two-body reactions, they all have$\lambda = 1$. Also, these reactions all have about the same value of the reduced mass, ranging from $12m_p/13$ to $15m_p/16$, while Z_1Z_2 only ranges from 6 for reaction i to 7 for reaction vi, so the Coulomb suppression factor and hence the rate factor R is smallest for reaction vi, but not overwhelmingly so. We will take the Coulomb barriers of these reactions to be a compromise, calculated by taking $Z_1Z_2 = 6.5$ and $\mu = m_p$. At any given temperature, the exponent in eq (157)

$$B(E_T) = \exp\left(-\frac{E_T}{k_BT} - \frac{C}{\sqrt{E_T}}\right) = \exp\left(-3\left(\frac{\pi Z_1 Z_2 e^2\sqrt{\mu}}{\hbar\sqrt{2k_BT}}\right)^{2/3}\right) \tag{157}$$

for the effective Coulomb barrier is thus larger than for the proton–proton chain by a factor $6.5^{2/3}2^{1/3}$ = 4.4. At a temperature of 10^7 K, the Coulomb barrier in the CNO cycle produces a suppression factor $\exp(-4.4\times 15.7) \simeq 10^{-30}$. But due to the extreme slowness of the weak interaction processes, such as the first reaction in the proton–proton chain, the CNO cycle can compete with the proton–proton chain at any temperature.

The power of temperature in Eq. (157) is larger than for the proton–proton chain by this same factor 4.4, so at $T \sim 10^7$ K we have in $(k_B T)^\nu$ that $\nu \sim 22$, and somewhat less at higher temperatures. λ in ρ^λ remains as discussed before $\lambda = 1$.

There are several alternative finales. Instead of step vi, the ^15N nucleus can undergo the reaction $^1H +^{15} N \to^{16} O + \gamma$, followed by $^1H +^{16} O \to^{17} F + \gamma$ and $^{17}F \to^{17} O + e^+ + \nu$. After that, there are again two possibilities: either $^1H +^{17} O \to^{14} N +^4 He$, or else $^1H +^{17} O \to^{18} F + \gamma$ followed by $^{18}F \to^{18} O + e^+ + \nu$ and $^1H +^{18} O \to^{15} N +^4 He$. In all cases the net effect is that four protons turn into a ^4He nucleus plus two positrons and two neutrinos, with the CNO catalysts always replenish to their original abundances.