Introduction to Astrophysics
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1 Homework 1

Exercise1

From the Theorem 1 on page 3, formula (10) use the inequality

$$P_c \geqslant \frac{GM^2}{8\pi R^4} \tag{1}$$

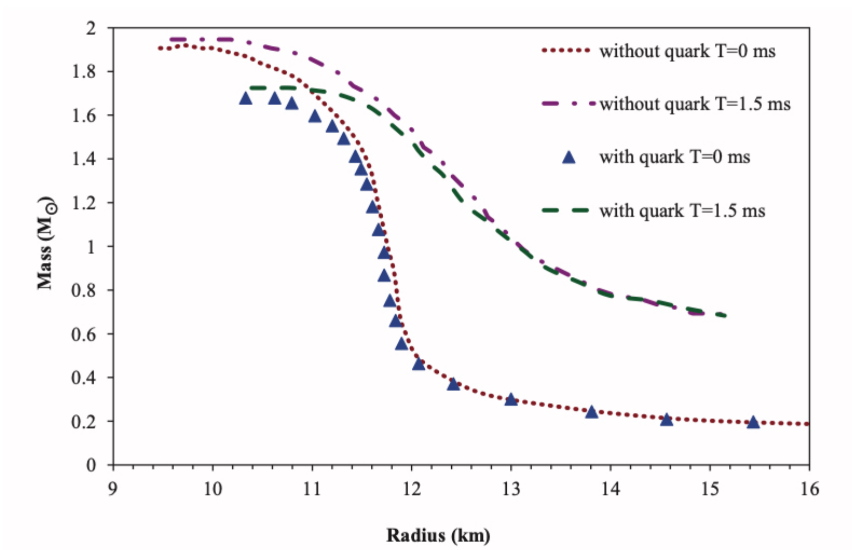

Figure 1: Mass-Radius relationship for static (T=0 ms) and rotating (T=1.5 ms) neutron stars with and without presence of quark matter inside the core (T is the period in miliseconds)

a) to estimate a lower bound for the pressure at the center of the Sun. Give the result in $atmospheres$ and $Newtons/m^2$.

b) Derive from (1) a general expression for a star of mass M and radius R in terms of $M_\odot$ and $R_\odot$, the mass and radius of the sun respectively.

c) Take a look at https://en.wikipedia.org/wiki/Main_sequence and in particular the Table of main-sequence stellar parameters: calculate the pressure inside the massive star BI 253 and also inside the dwarf star 61 Cygni A.

d) Look at the Figure 1. Upper limit masses for neutron stars from observations (GRBs, Grav waves, MMA) are about 2 - 2.2 $M_\odot$ and radius about 10 -12 km. What would be a lower bound for the pressure at the center of the star?

Exercise2

From the Virial theorem we learned that in the case of the Newtonian potential

$$\bar{T} = -\frac{1}{2}\bar{U} \tag{2}$$

a) What is the relationship between the time averaged kinetic energy and potential energy for the harmonic oscillator ($U(r) = \frac{1}{2}\kappa r^2$, where κ is the elastic constant of the oscillator)?

b) Assume that two different masses m_1 and m_2 follow the same path in the same potential energy. Prove that the relationship between the times spent in the path and the masses is:

$$\frac{t_2}{t_1} = \sqrt{\frac{m_2}{m_1}} \tag{3}$$

c) Assume now that two equal masses follow the same path but in a potential that differ by a constant factor. Let's call the time for the mass moving in the potential U_1, t_1 and t_2 the time for the mass moving in the potential U_2. Prove that the relationship between the times and the potential is:

$$\frac{t_2}{t_1} = \sqrt{\frac{U_1}{U_2}} \tag{4}$$