Introduction to Astrophysics
Fall 2021

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1 Homework 2

Exercise1

Show explicit using the formulas (33) from the Lecture notes from August 30 that if the factor $\Gamma = 4/3$ the potential energy is equal to the thermal energy. What is the value of the total energy in that case? Explain from this why for the star to be stable (not to explode) the potential energy has to be larger then the thermal one.

Exercise 2

We showed in the class of 9-01-2021 that the energy-momentum tensor for an electromagnetic field can be written

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix} = -F_{\nu\mu}$$

The trace of this tensor (the sum of the diagonal entries) is zero. This is a general result: the trace of the energy momentum tensor for an electromagnetic field is always zero.

In the case of a fluid with a general energy density ρ and pressure p the energy momentum tensor is represented by a tensor (written in the reference frame co-moving with the fluid)

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}, \tag{1}$$

In the case of a gas of photons (which have zero mass) the equation of state of the fluid $p(\rho)$ is given by

$$p(\rho) = \frac{1}{3}\rho \tag{2}$$

Prove that the energy momentum tensor of such a fluid is consistent with that of an electromagnetic field, i.e. its trace is zero.