Introduction to Astrophysics
Fall 2021
Homework 5

Mario C Díaz

The material for these exercises have been adapted from Carroll, Bradley W.; Ostlie, Dale A., An Introduction to Modern Astrophysics, Cambridge University Press.

Exercise 1

Show that the equation for hydrostatic equilibrium, Eq. (6) from the lecture notes, can also be written in terms of the optical depth τ, as

$$\frac{dP}{d\tau} = -\frac{G\rho\mathcal{M}}{\kappa r^2}.$$

Hint: show first that the integral equation (113) from Lecture notes can be written in a differential form for $d\tau$.

This form of the equation is often useful in building model stellar atmospheres.

Exercise 2

The purpose of this exercise is to estimate the hydrogen-burning lifetimes of stars near the lower and upper ends of the main sequence. The lower end of the main sequence occurs near $0.072M_\odot$, with $T_e = 10^{3.23}$ and $L/L_\odot = 10^{-4.3}$. On the other hand, an $85M_\odot$ star near the upper end of the main sequence has an effective temperature and luminosity of $Te = 10^{4.705}$ and $L/L_\odot = 10^{6.006}$, respectively. Assume that the $0.072M_\odot$ star is entirely convective so that, through convective mixing, all of its hydrogen, rather than just the inner 10%, becomes available for burning.

Hints:

1) First compute the star luminosity for each case (the $0.072M_\odot$ and the $85M_\odot$ stars) in Watts.

2) Assume then a pure hydrogen composition for simplicity, and that the entire star participates in the energy generation, the amount of energy released in the conversion of hydrogen to helium for a star of mass M is $E = 0.007Mc^2$ (calculate it in Joules).

(The 0.007 factor comes from the considerations below)

The simplest isotope of hydrogen is composed of one proton and one electron and has a mass of $m_H = 1.00782503214$ u. This mass is actually very slightly less than the combined masses of the proton and electron taken separately. In fact, if the atom is in its ground state, the exact mass difference is 13.6 eV, which is just its ionization potential. Since mass is equivalent to a corresponding amount of energy, and the total mass–energy of the system must be conserved, any loss in energy when the electron and proton combine to form an atom must come at the expense of a loss in total mass. Similarly, energy is also released with an accompanying loss in mass when nucleons are combined to form atomic nuclei. A helium nucleus, composed of two protons and two neutrons, can be formed by a series of nuclear reactions originally involving four hydrogen nuclei (i.e., $4H \rightarrow He+$ low mass remnants). Such reactions are known as fusion reactions, since lighter particles are "fused" together to form a heavier particle. (Conversely, a fission reaction occurs when a massive nucleus is split into smaller fragments.) The total mass of the four hydrogen atoms is 4.03130013 u, whereas the mass of one helium atom is $m_{He} = 4.002603$ u. Neglecting the contribution of low-mass remnants such as neutrinos, the combined mass of the hydrogen atoms exceeds the mass of the helium atom by $m = 0.028697$ u, or 0.7%

3) Then calculate the hydrogen-burning lifetime as

$$t_X = \frac{E_X}{L_X} \tag{1}$$

where X refers to either the $0.072M_\odot$ or the $85M_\odot$ one. You should get an answer in seconds. Convert it to years.

Exercise 3

Using the Stefan-Boltzman as in equation (112) from the lecture notes and the luminosities estimated in the previous exercise, calculate the radii of a $0.072M_\odot$ and of the $85M_\odot$ star. Give the answer in terms of $R_\odot$. What is the ratio of their radii?

The following exercise is only for the graduate students who are taking the PHYS 6381 class.

Exercise 4

(a) Estimate the so called Eddington luminosity (equation (125) in the lecture notes, the limit in the Eddington instability (124)) $\kappa(R)L = 4\pi GcM$ of a $0.072M_\odot$ star and compare your answer to the main-sequence luminosity given in the previous exercise. Assume $\kappa = 0.001m^2kg^{-1}$. Is

radiation pressure likely to be significant in the stability of a low-mass main-sequence star?
(b) If a $120M_\odot$ star forms with $Te = 10^{4.727}$ and $L/L\odot = 10^{6.252}$ estimate its Eddington luminosity assuming the opacity is due to electron scattering. Compare your answer with the actual luminosity of the star.