PHYS 5310
CLASSICAL MECHANICS - 2024

HOMEWORK 4

Exercise 1.

Find the equations of motion for a particle in the field:

a)

$$U(x) = -\frac{U_0}{\cosh^2 \alpha x}$$

b)

$$U(x) = U_0 \tan^2 \alpha x$$

Exercise 2.

Find the equations of motion for a particle in the field $U(x) = -Ax^4$ if its energy is 0.

Exercise 3.

Consider how the equations of motion change when you "add" a small quantity $\delta U(x)$ to the field $U(x)$ where there are no turning points.
Use this consideration to find the change in:

$$U(x) = \frac{m\omega^2 x^2}{2}$$

when you add $\delta U(x) = \frac{m\alpha x^3}{3}$.

Exercise 4.

Find how the finite period of motion of a particle in the field $U(x)$ changes when a small quantity $\delta U(x)$ is added to it. Use your result to study the change in the finite period of a particle in field

$$U(x) = \frac{1}{2} m\omega^2 x^2$$

when it is changed by a small quantity

$$\delta U(x) = \frac{1}{4} m\beta x^4.$$

Exercise 5.

Integrate the equations of motion for a particle in the central field assuming different values of the energy (i.e. $E <, >, = 0$) and relationships between momentum and α.

$$U(r) = -\frac{\alpha}{r^2} \qquad \alpha > 0.$$

Exercise 6.

Find the equations of motion and trajectories of a particle in the field

$$U(r) = \begin{cases} -V, & \text{if } r < R, \\ 0, & \text{if } r > R. \end{cases} \tag{1}$$

for different values of energy and momentum. This potential is called the rectangular spherical potential well.