PHYS 5310
CLASSICAL MECHANICS - 2024

HOMEWORK 6

Exercise 1.
Show that

$$x(t) = A\cos\omega t + B\sin\omega t. \quad (1)$$

can be written as

$$x(t) = \mathcal{A}\cos(\omega t + \alpha) \quad (2)$$

where

$$\mathcal{A} = \sqrt{A^2 + B^2} \quad (3)$$

and

$$\tan\alpha = -\frac{B}{A} \quad (4)$$

Hint: use that $\cos(\gamma + \delta) = \cos\gamma\cos\delta - \sin\gamma\sin\delta$.

Exercise 2.
a) Show that using equation (6) from Lesson 6, the formula (12) for the energy of a harmonic oscillator can be written as formula (13).
b) Show that with the definition from equation (15) also in Lesson 6 formula (14) is equivalent to formula (9).

Exercise 3.
If the initial position and initial velocity of a harmonic oscillator are respectively x_o and v_o write the expressions (10 and (11) for the amplitude an initial phase of the oscillations.

Exercise 4.
a) In the Example of Lesson 6 show that equations (18) and (19) lead to write the final Lagrangian of the sliding pendulum as expression (22).
b) Solving equation (26) to obtain the result (27).
c) By explicitly calculating both components of the center of mass of the system show explicitly that (27) express the fact that the horizontal component of the center of mass of the system does not move.
d) Calculate the total energy of the system and show that the energy is given by (30).

e) Show that for small oscillations the energy is given by (32). Find the frequency of small oscillations.

Exercise 5.

Show that the trajectory of mass m_2, in the sliding pendulum, is an ellipse with horizontal semi-axis $lm_1/(m_1+m_2)$ and vertical semi-axis l.

Exercise 6.

find the frequency of small oscillations in the field

$$U(x) = V\cos(\alpha x) - Fx. \tag{5}$$

Exercise 7.

Show that using a particular solution for the inhomogeneous equation (36) in Lesson notes 6 of the form $x_1(t) = b\cos(\psi t + \beta)$ the general solution for it is of the form

$$x(t) = A\cos(\omega t + \alpha) + \left[\frac{f}{m(\psi^2 - \omega^2)}\right]\cos(\psi t + \alpha) \tag{6}$$

Exercise 8.

Calculate C^2 using the definition of equation (43) calculating the product of C and its complex conjugate $C*$ proving then the result (44). Why then, the allowed range of values of $|C|$ are as determined by equation (45)?

Exercise 9.

Show that if

$$x_1(t) = \mu_1^{t/T} H_1(t), \qquad\qquad x_2(t) = \mu_2^{t/T} H_2(t), \tag{7}$$

where $H_1(t)$ and $H_2(t)$ are periodic functions of t of period T are solutions for the equation

$$\frac{d^2x}{dt^2} + \omega(t)x = 0 \tag{8}$$

when multiplying $\ddot{x}_1 - \omega^2(t)x_1 = 0$ by x_2 and $\ddot{x}_2 - \omega^2(t)x_2 = 0$ by x_1 you obtain that:

$$\dot{x}_1 x_2 - x_1 \dot{x}_2 = constant \tag{9}$$

Exercise 10.

How are μ_1 and μ_2 related? Show that for any function x_1 and x_2 of the form (6) equation (8) will be multiply by a factor $\mu_1\mu_2$ when t is replaced by $t+T$ where T is the period of the function. Conclude that for equation (8) to hold

$$\mu_1\mu_2 = 1$$

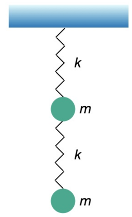

Figure 1: A 2 degree of freedom oscillation system

Exercise 11.
Prove formula (107).

Exercise 12.
Find the free oscillations of the system represented in the Figure 1.

Exercise 13.
a) Find the anharmonic oscillations of the pendulum in the system represented in the Figure 2. The suspension of the pendulum moves along the circumference.
b) Determine its position of stable equilibrium if its support point oscillates with a frequency $\omega \gg \sqrt{g/l}$.

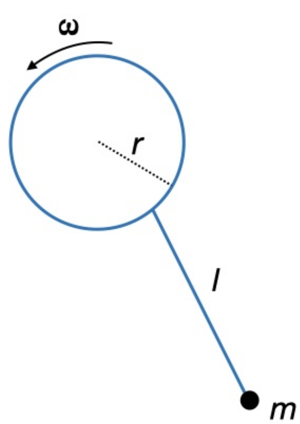

Figure 2: A pendulum with its suspension point moving along a circumference.