Lesson 2

Probability and Statistics

Mean and standard deviation

Let’s have n measurements of a quantity, a

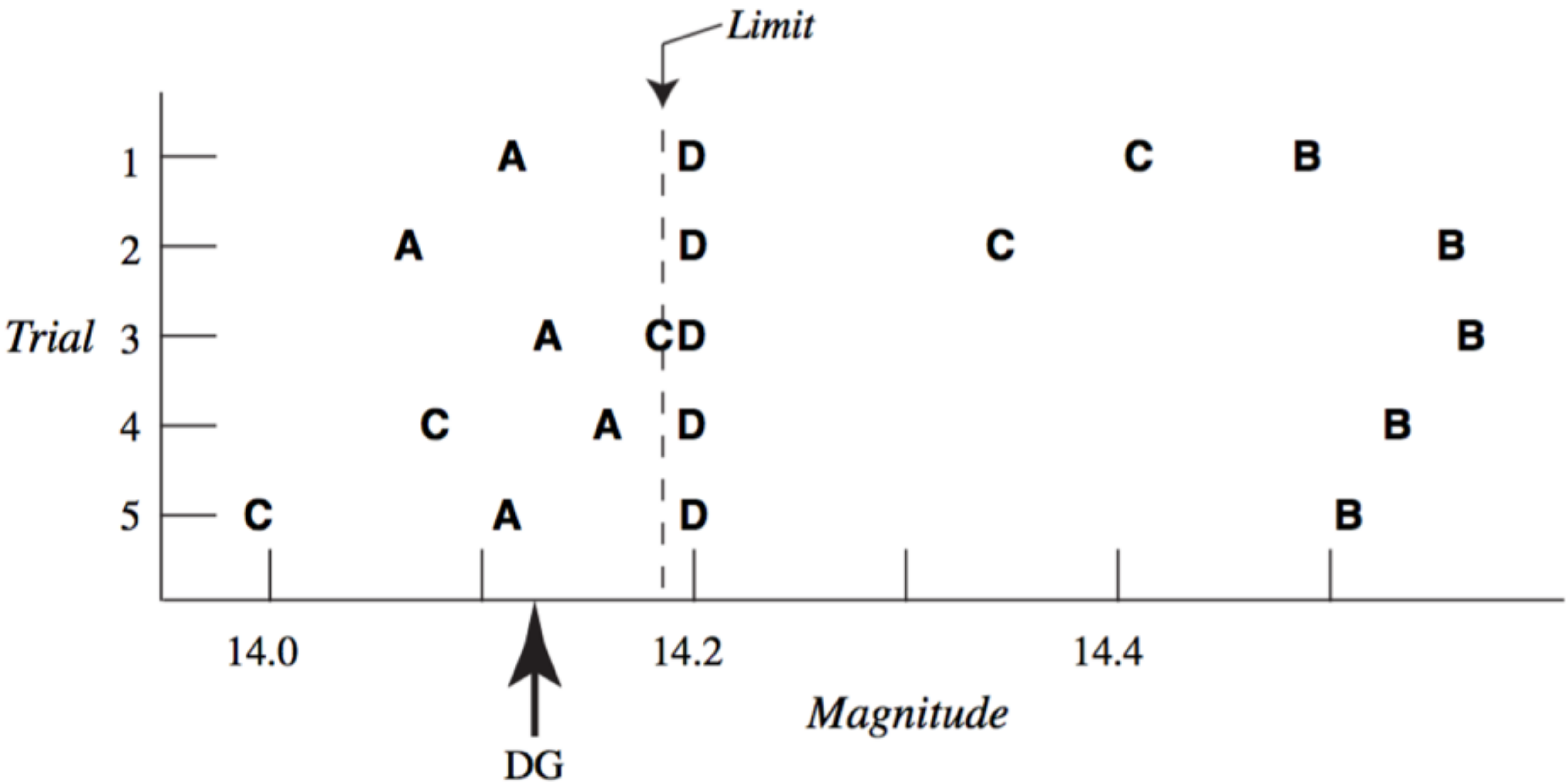
$$A = \frac{1}{n}\sum_{i=1}^{n} a_i$$

A is the mean

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(a_i - A)^2}$$

Variance and estimated standard deviation

$$Var(a) = \frac{1}{n}\sum_{i=1}^{n}(a_i - A)^2 \qquad \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(a_i - A)^2}$$


Limit
Trial
1
2
3
4
5
A D C B
A D C B
A CD B
C A D B
C A D B
14.0
14.2
14.4
Magnitude
DG

Example

Astronomer	A	B	C	D
Trial 1	14.115	14.495	14.386	14.2
Trial 2	14.073	14.559	14.322	14.2
Trial 3	14.137	14.566	14.187	14.2
Trial 4	14.161	14.537	14.085	14.2
Trial 5	14.109	14.503	13.970	14.2
Mean	14.119	14.532	14.190	14.2
Deviation from truth	−0.004	+0.409	+0.067	+0.077
Spread	0.088	0.071	0.418	0
σ	0.033	0.032	0.174	0
s	0.029	0.029	0.156	0
Uncertainty of the mean	0.013	0.013	0.070	(0.05)
Interpretation	Evacuate	Stay	Uncertain	Uncertain
Accuracy?	Accurate	Inaccurate	Accurate	Inaccurate
Precision?	Precise	Precise	Imprecise	Imprecise

- Precision is very different from accuracy.
- Systematic errors.
- Random errors could be quantify looking at the spread of the values.
- Un certainty got like $s/\sqrt{N}$.

Significant Digits

- Astronomer C has measured 14.190 with uncertainty $0.156/\sqrt{5} = 0.070$. The result should be quoted as 14.19
- A 14.12 ± 0.013
- B 14.53 ± 0.013
- C 14.19 ± 0.07

Population	Sample	Better sample
1000 colored marbles mixed in a container: 500 red, 499 blue, 1 purple	5 marbles drawn at random from the container	50 marbles drawn at random
The luminosities of each star in the Milky Way galaxy (about 10^{11} values)	The luminosities of each of the nearest 100 stars (100 values)	The luminosities of 100 stars at random locations in the galaxy (100 values)
The weights of every person on Earth	The weights of each person in this room	The weights of 100 people drawn from random locations on Earth
The outcomes of all possible experiments in which one counts the number of photons that arrive at your detector during one second from the star Malificus	The outcome of 1 such experiment	The outcomes of 100 such experiments

Table 2.3. *Employee salaries at Astroploitcom*

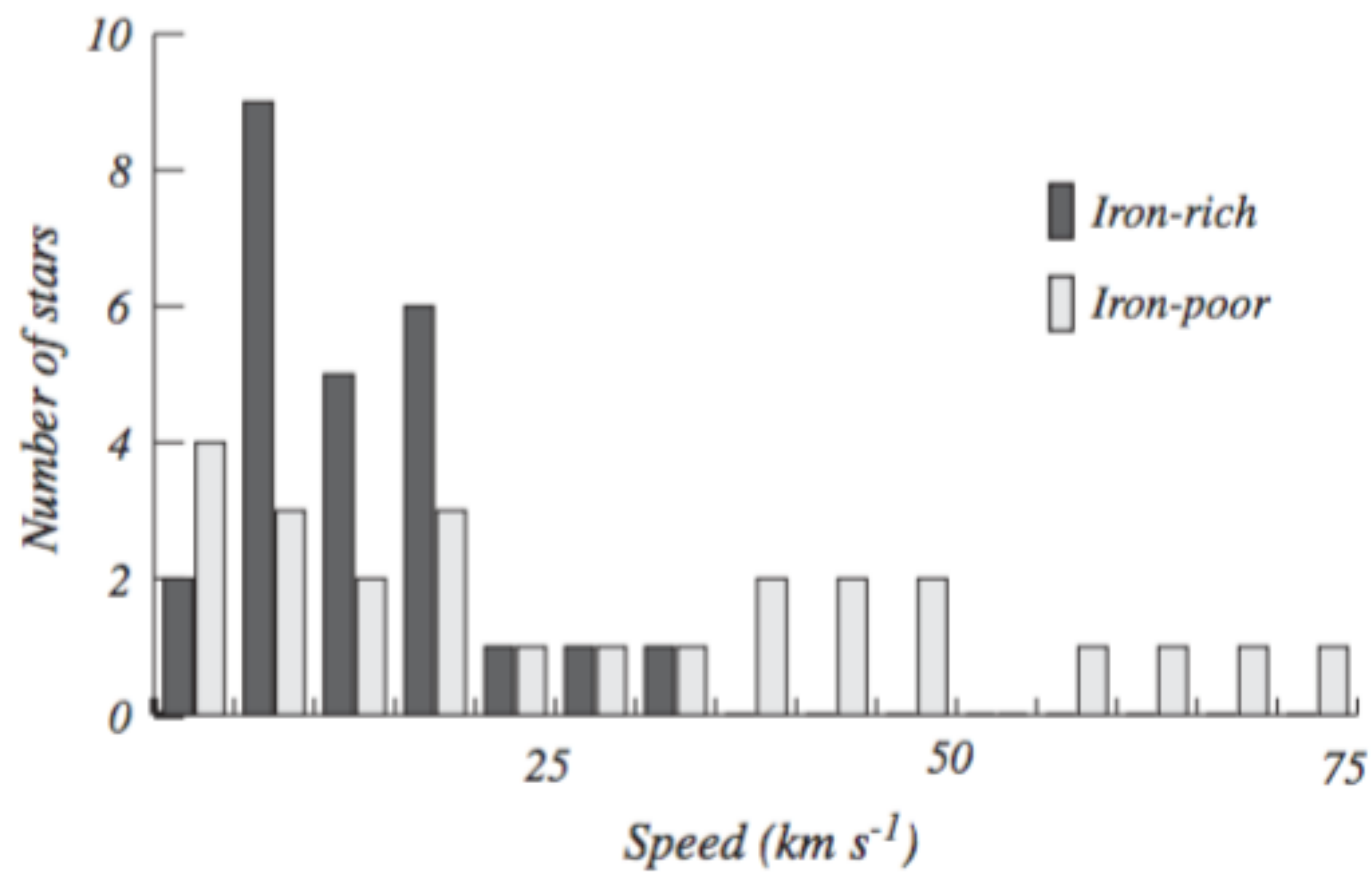
Job title (number of employees)	Salary in thousands of dollars
President (1)	2000
Vice president (1)	500
Programmer (3)	30
Astronomer (4)	15

We know the mean, we can also calculate the median $\mu_{1/2}$ is such $n(x_i \leq \mu_{1/2})=n(x_i \geq \mu_{1/2})\approx M/2$.
The mode is the most common or frequent value.
How much is the mean?
How much is the median?
The mode?

$300,000, $30,000, $15,000

Table 2.4. *Speeds perpendicular to the Galactic plane, in km s^{-1}, for 50 nearby solar type stars*

Group A: 25 Iron-rich stars					Group B: 25 Iron-poor stars				
0.5	7.1	9.2	14.6	18.8	0.3	7.9	16.8	35.9	48.3
1.1	7.5	10.7	15.2	19.6	0.4	10.0	18.1	38.8	55.5
5.5	7.8	12.0	16.1	24.2	2.5	10.8	23.1	42.2	61.2
5.6	7.9	14.3	17.1	26.6	4.2	14.5	26.0	42.3	67.2
6.9	8.1	14.5	18.0	32.3	6.1	15.5	32.1	46.6	76.6

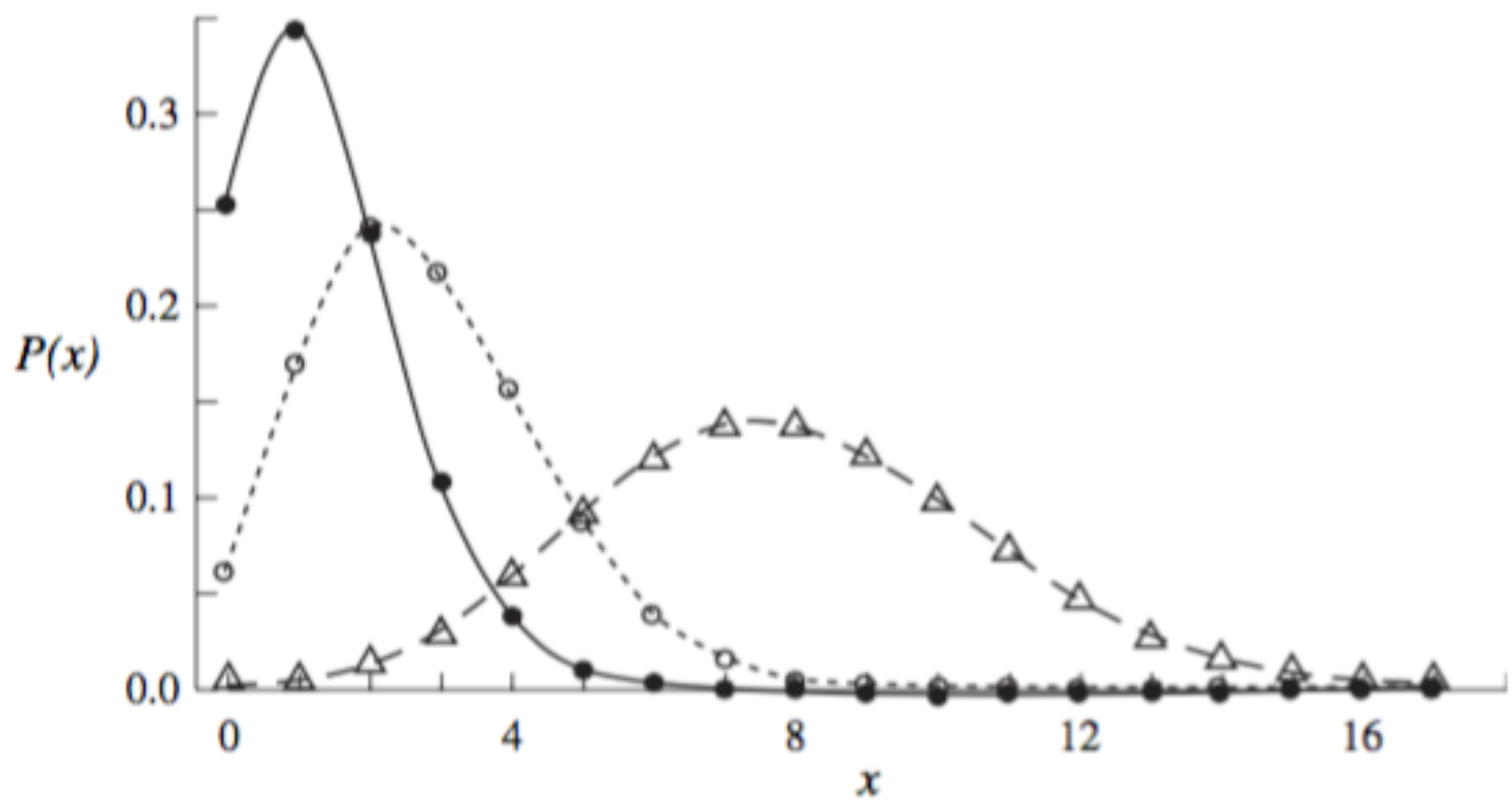


There are cases in which the random variable, x, is the number of events counted in a unit time: the number of raindrops hitting a tin roof in 1 second, the number of photons hitting a light meter in 10 seconds, or the number of nuclear decays in an hour. For counting experiments where non-correlated events occur at an average rate, l, the probability of counting x events in a single trial is

$$P_p(x, \mu) = \frac{\mu^x}{x!} e^{-\mu}$$

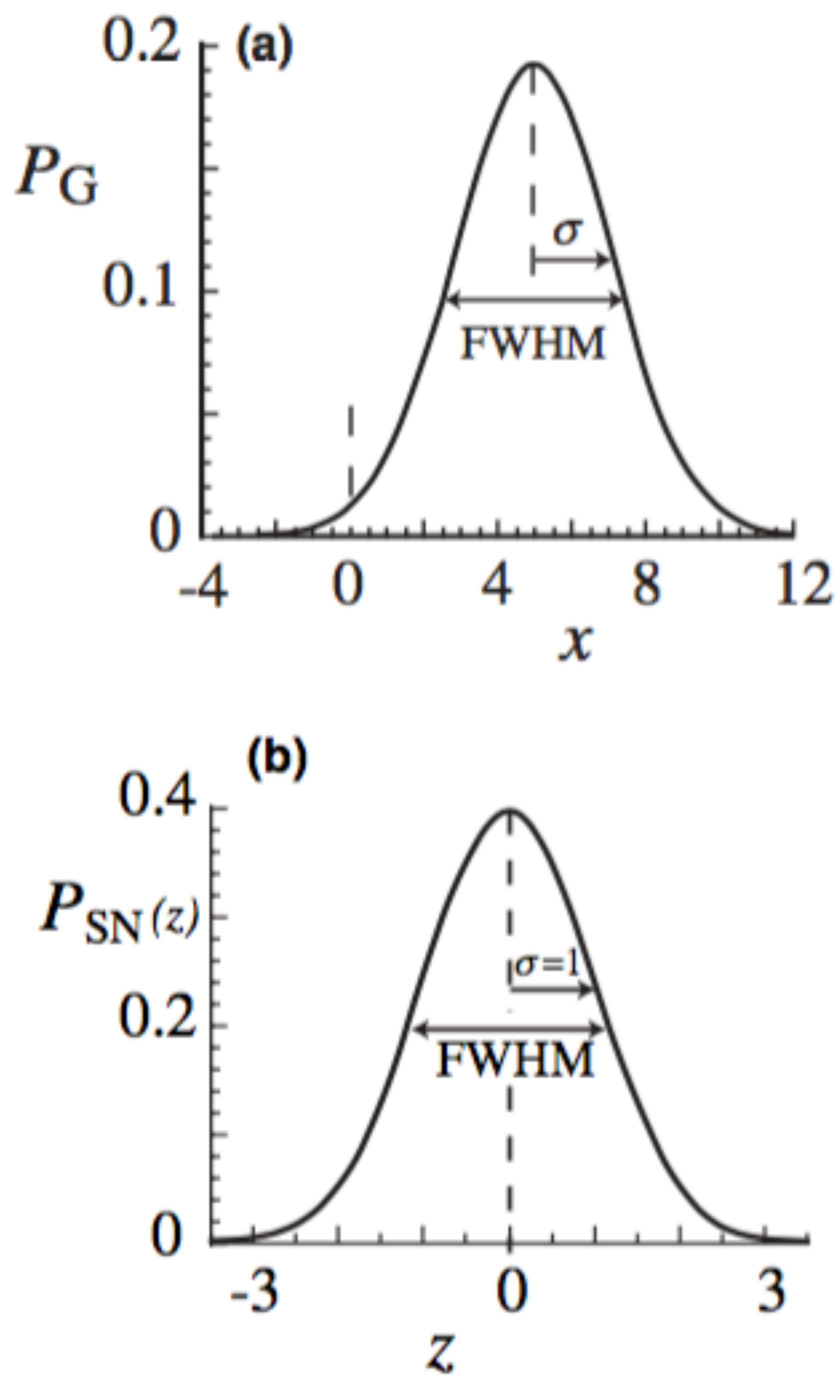
$$\sigma^2 = \mu$$

$$\textit{Fractional uncertainty in counting N events} = \frac{\sigma}{\mu} \approx \frac{1}{\sqrt{N}}$$



0.3
0.2
0.1
0.0
P(x)
0
4
8
12
16
x

$$P_G(x, \mu, \sigma)\mathrm{d}x = \frac{\mathrm{d}x}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$



Mean 5,
σ = 2.1
FWHM 2.354 σ

An example from detecting gravitational waves

The era of gravitational wave astronomy:

how many black holes are out there?

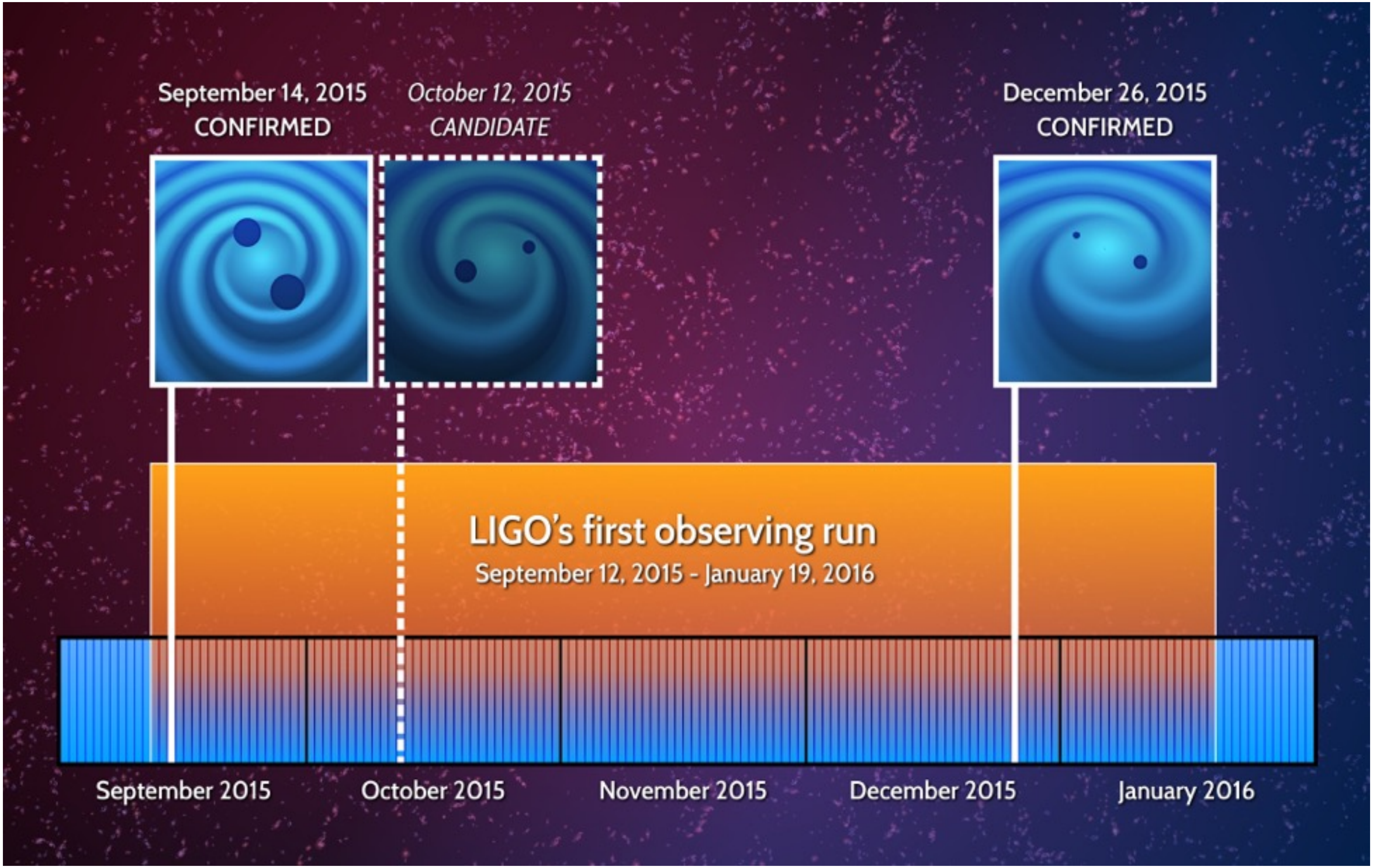
Mario Díaz
for the LVC Collaboration

Center for Gravitational Wave Astronomy
UTRGV

Washington DC, 2-11-2016

Detection

- On September 14, 2015 at 09:50:45 UTC, the LIGO Hanford, WA, and Livingston, LA, observatories detected the coincident signal GW150914.
- The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203,000 years, equivalent to a significance greater than 5.1σ.



September 14, 2015
CONFIRMED
October 12, 2015
CANDIDATE
December 26, 2015
CONFIRMED
LIGO's first observing run
September 12, 2015 - January 19, 2016
September 2015
October 2015
November 2015
December 2015
January 2016

A long road

- Einstein's GR theory 1915.
- Gravitational waves, 1916.
- 1918, time dependance of the quadrupolar mass tensor as the source.
- 1920-1970 doubts, uncertainties and little else.
- 1970-1990 resonant bars.

The challenge

Imagine this distance: around the world 100,000 million times (the total is 3, 840 billions of kilometers, or a million times the distance from Earth to Neptune). Take two small objects separated by this distance. A strong gravitational wave will modify it less than the thickness of a human hair. There is less than a few tenths of a second to measure this. And we don't know if this event will happen next month, next year or in thirty years.

(testimony from Anthony Tyson to the subcommittee of Science from the Committee of Science, Space and Technology in the House of Representatives when the construction of LIGO was under consideration, March 13 1991).

But they were built!

Some more recent history

- LIGO is the largest single enterprise undertaken by NSF, with capital investments of nearly $300 million and operating costs of more than $30 million/year.

- LIGO has gathered data from 2002 to 2009 in several observational runs (S1 to S5).

- July 2009 - October 2010 S6: Twice improved original design sensitivity for what was called Enhanced LIGO.

- Since then drastic upgrade in new technology to improve the sensitivity one order of magnitude over initial LIGO:“Advanced LIGO”.

- It was scheduled to start operating in what has been dubbed O1 ~ September 2015.

LIGO Scientific Collaboration

Abilene Christian University
Albert-Einstein Institut
Andrews University
American University
California Institute of Technology
California State Univ., Fullerton
Canadian Inst. Th. Astrophysics
Carleton College
College of William and Mary
Columbia University
Embry-Riddle Aeronautical Univ.
Eötvös Loránd University
Georgia Institute of Technology
Goddard Space Flight Center
Hobart & William Smith Colleges
ICTP-SAIFR
IndIGO
IAP-Russian Acad. of Sciences
Inst. Nacional Pesquisas Espaciais
Kenyon College
Korean Gravitational-Wave Group
Louisiana State University
Montana State University
Montclair State University
Moscow State University
National Tsinghua University
Northwestern University

Penn State University
Rochester Institute of Technology
Sonoma State University
Southern Univ. and A&M College
Stanford University
Syracuse University
Szeged University
Texas Tech University
Trinity University
Tsinghua University
Universitat de les Illes Balears
University of Alabama in Huntsville
University of Brussels
University of Chicago
University of Florida
University of Maryland
University of Michigan
University of Minnesota
University of Mississippi
University of Oregon
University of Sannio
Univ. of Texas-Rio Grande Valley
University of Washington
University of Wisconsin-Milwaukee
Washington State University
West Virginia University
Whitman College

Australian Consortium for Interferometric Gravitational Astronomy (ACIGA):
Australian National University, Charles Sturt University, Monash University, University of Adelaide, University of Melbourne, University of Western Australia
LIGO Laboratory: California Institute of Technology, Massachusetts Institute of Technology, LIGO Hanford Observatory, LIGO Livingston Observatory
German/British Collaboration for the Detection of Gravitational Waves (GEO600):
Cardiff University, Leibniz Universität Hannover, Albert-Einstein Institut, Hannover, King's College London, Rutherford Appleton Laboratory, University of Birmingham, University of Cambridge, University of Glasgow, University of Hamburg, University of Sheffield, University of Southampton, University of Strathclyde, University of the West of Scotland

El mexicano que participó en el descubrimiento de las ondas gravitacionales

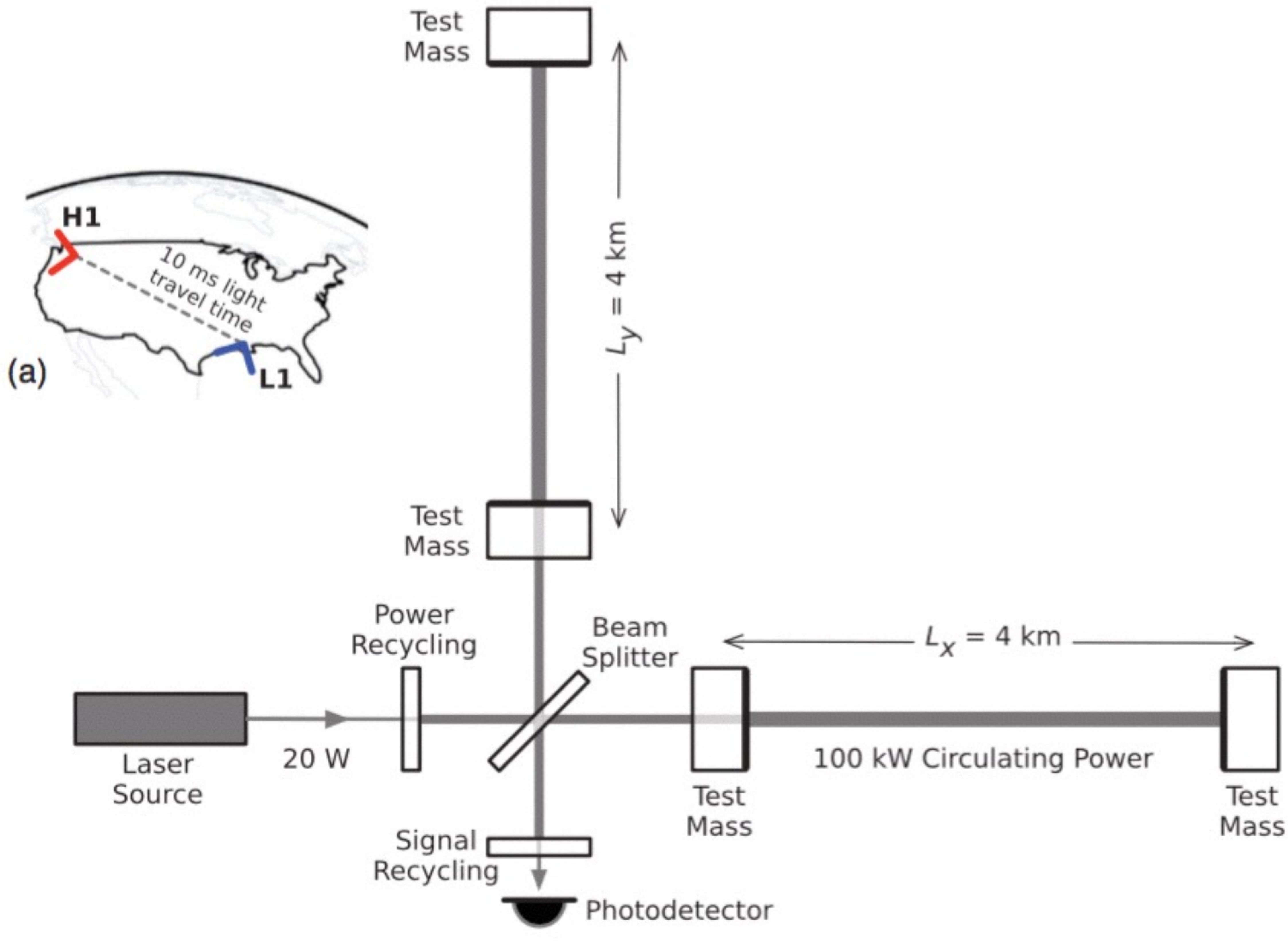
Twittear G+1 Me gusta Compartir

Por Verenise Sánchez

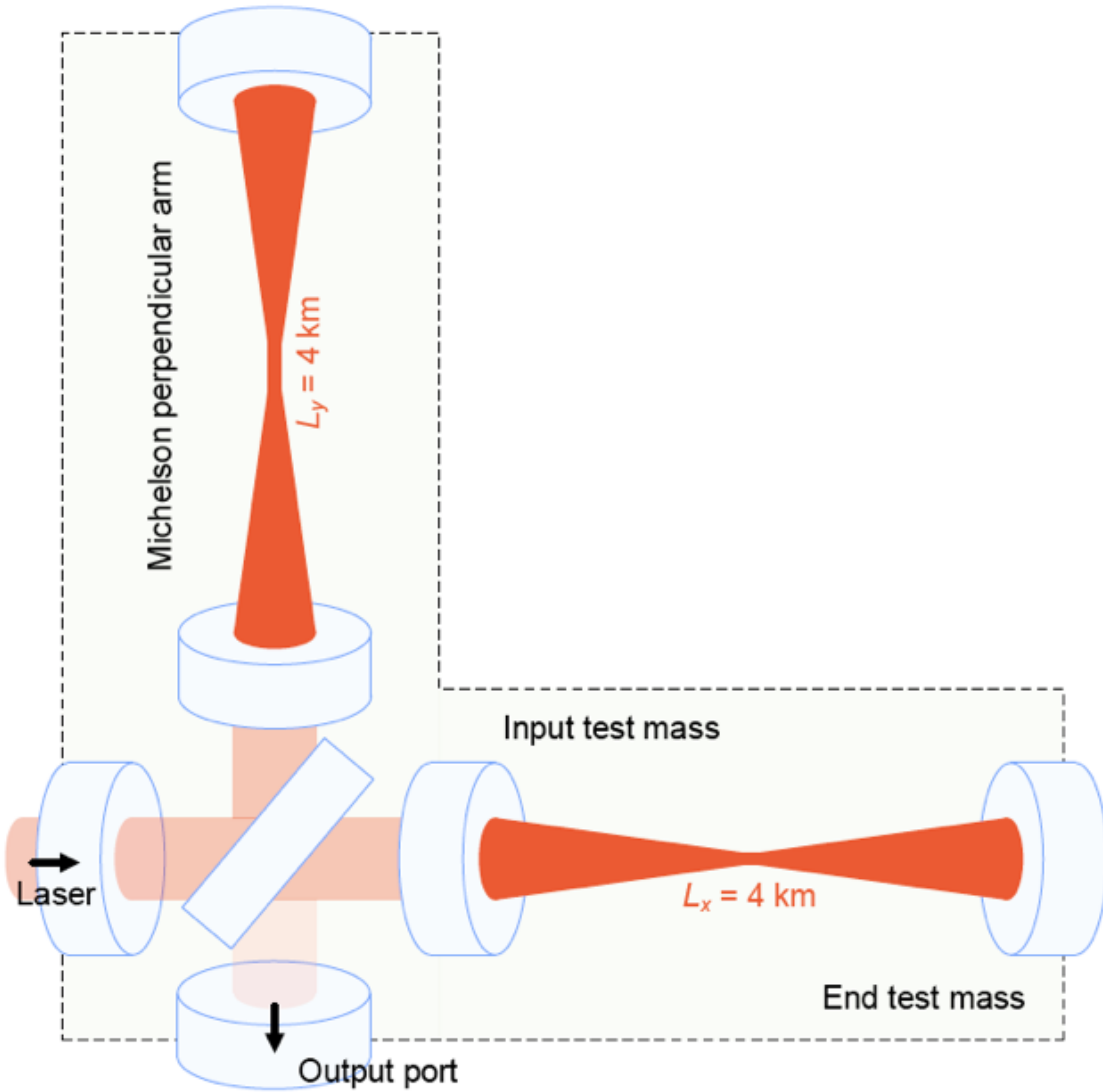
Ciudad de México. 23 de febrero de 2016 (Agencia Informativa Conacyt).- Eran las 9:40 de la mañana del 14 de septiembre de 2015, todo parecía normal en el segundo día de operación de la nueva fase del Observatorio de Ondas Gravitacionales por Interferometría Láser (LIGO, por sus siglas en inglés), narra Guillermo Adrián Valdés Sánchez, el científico mexicano que participa en dicho proyecto.

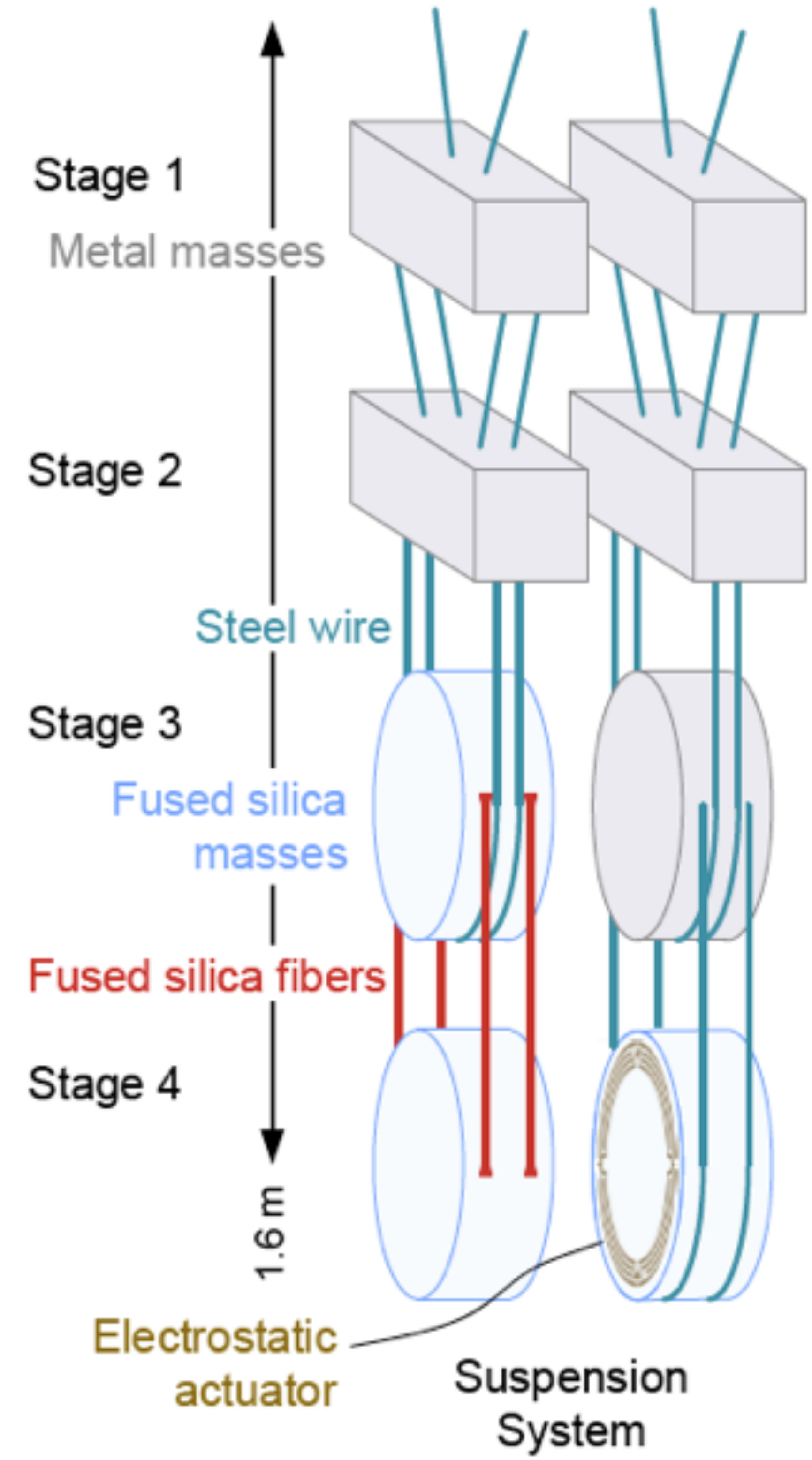
asesor y amigo, el doctor Mario Díaz.

http://www.conacytprensa.mx/index.php/sociedad/personajes/5632-el-mexicano-que-participo-en-el-hallazgo-de-la-primera-onda-gravitacional



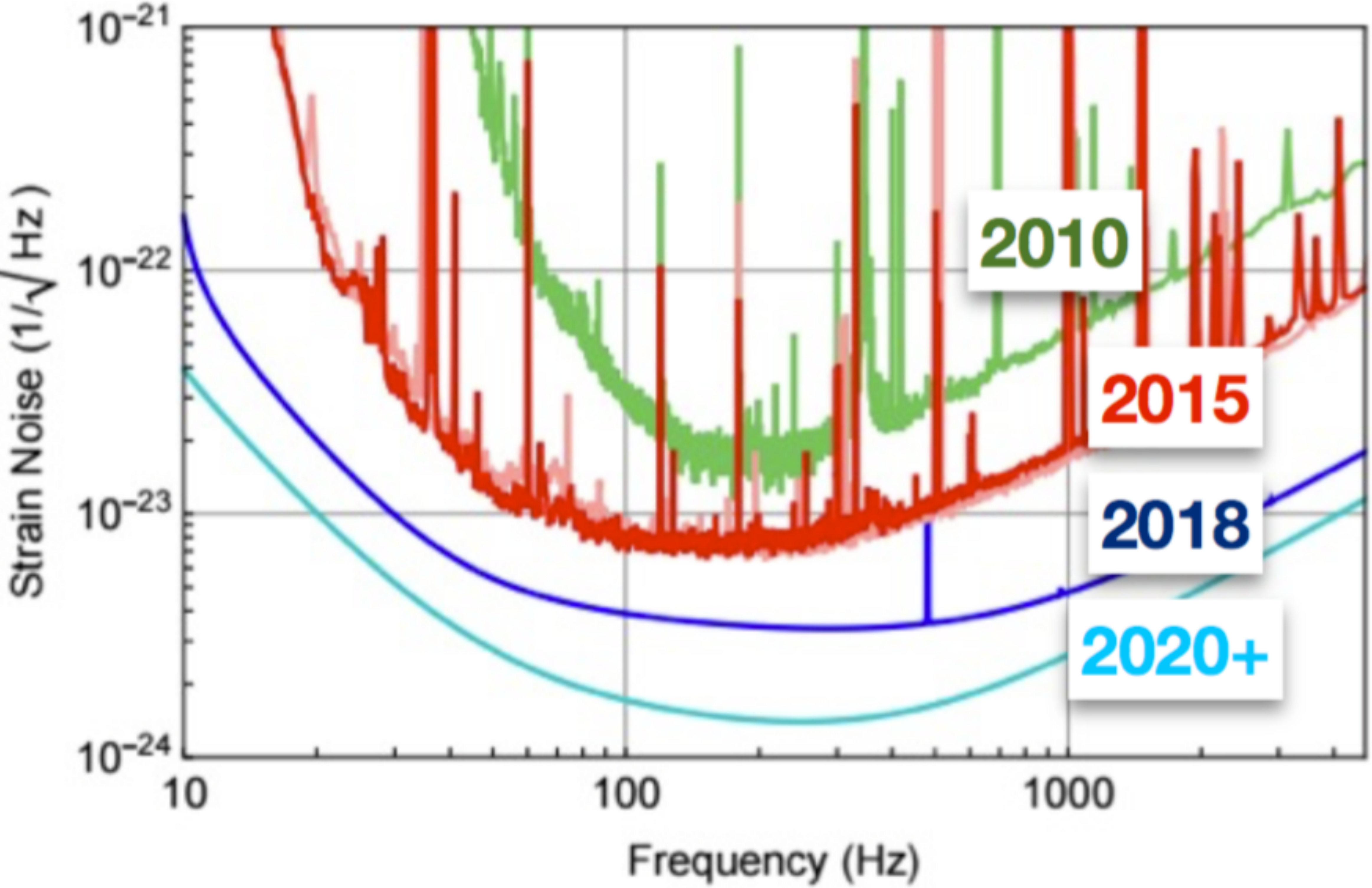
from Abbott et al. 2016, PRL 116, 061102



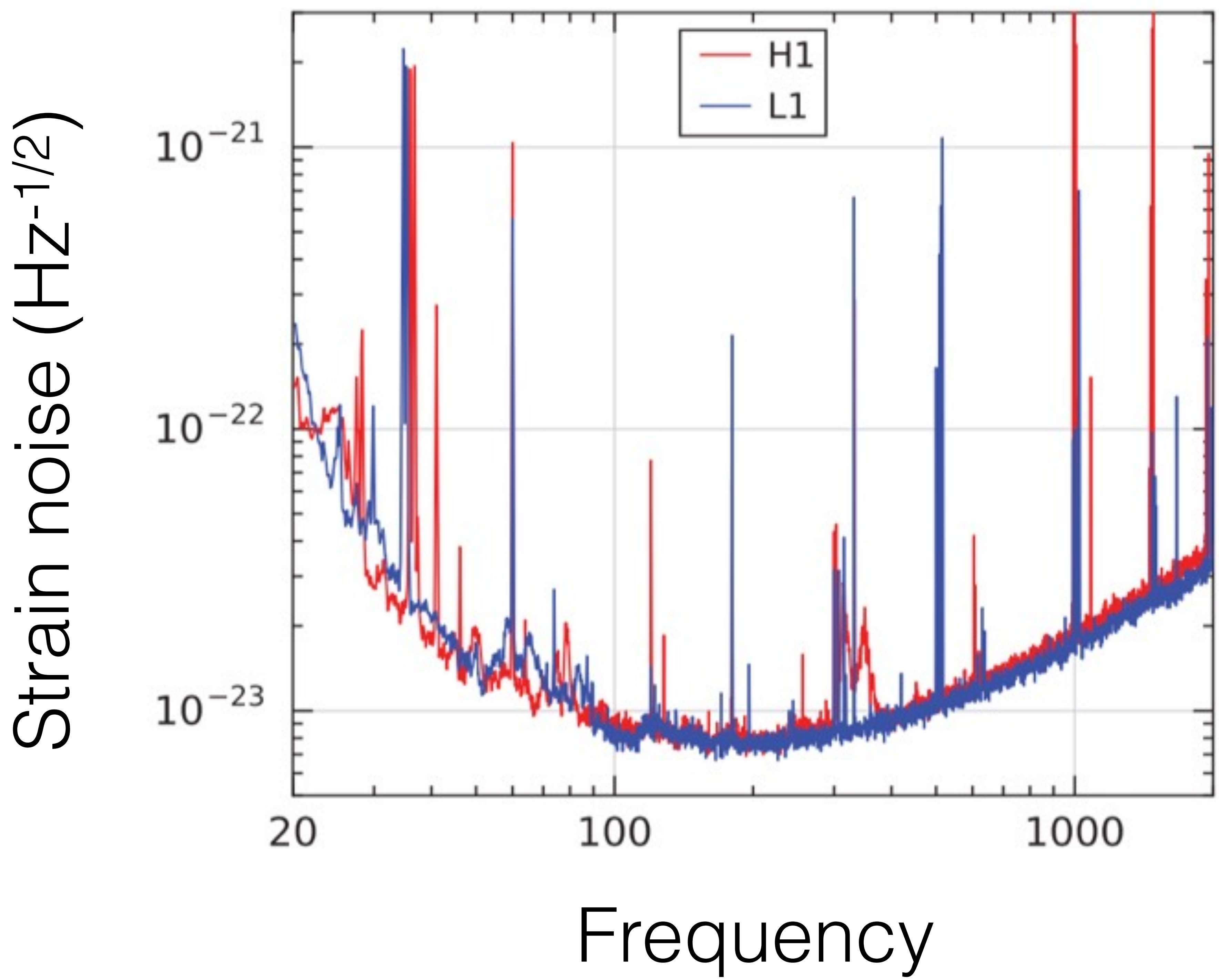


from Abbott et al. 2016, PRL 116, 061102

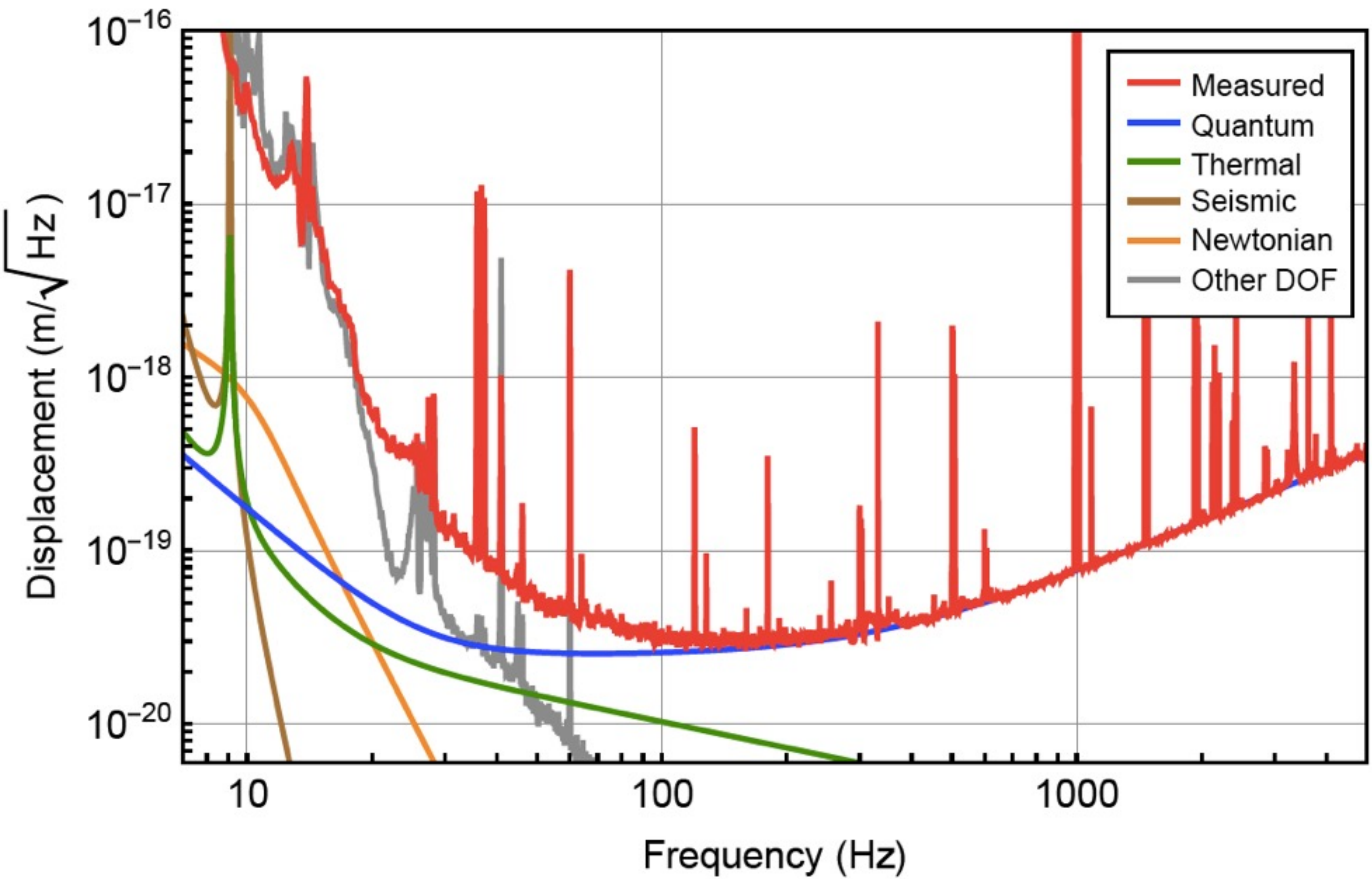
IFO performance



Credit: C. Pankow

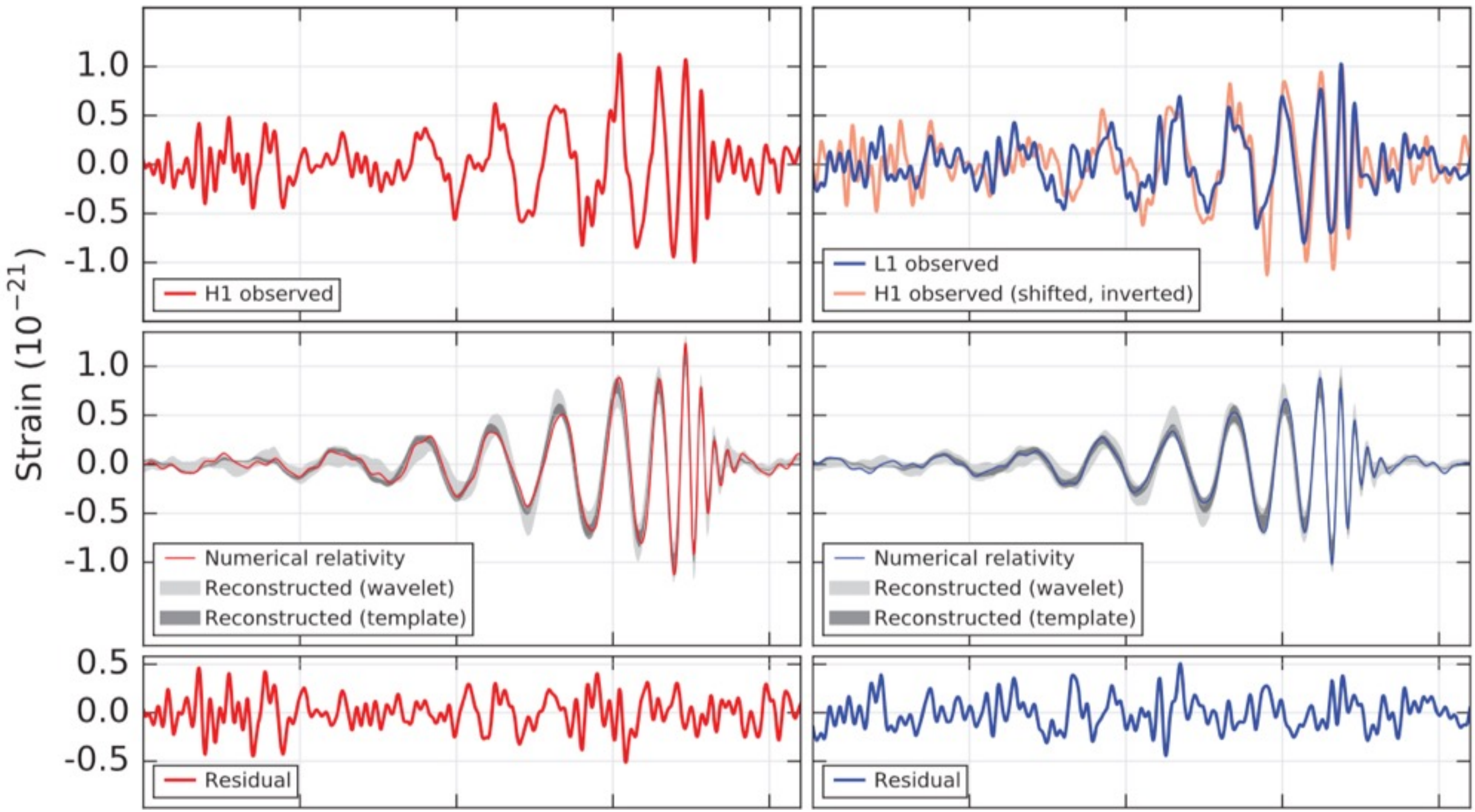


Narrow-band features include calibration lines (33–38, 330, and 1080 Hz), vibrational modes of suspension fibers (500 Hz and harmonics), and 60 Hz electric power grid harmonics.



Measured
Quantum
Thermal
Seismic
Newtonian
Other DOF
Displacement (m/√Hz)
Frequency (Hz)
10⁻¹⁶
10⁻¹⁷
10⁻¹⁸
10⁻¹⁹
10⁻²⁰
10
100
1000

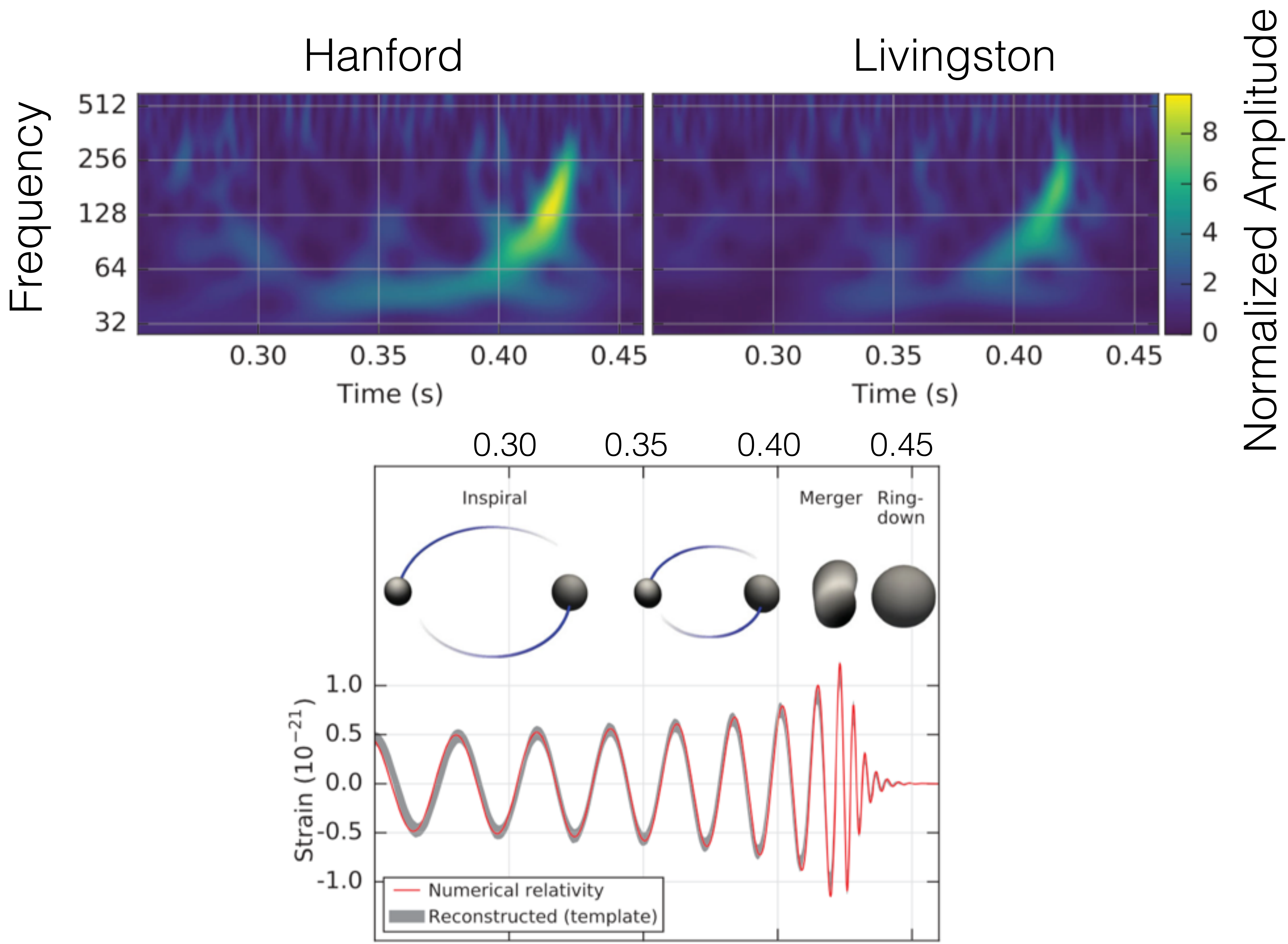
Hanford Livingston



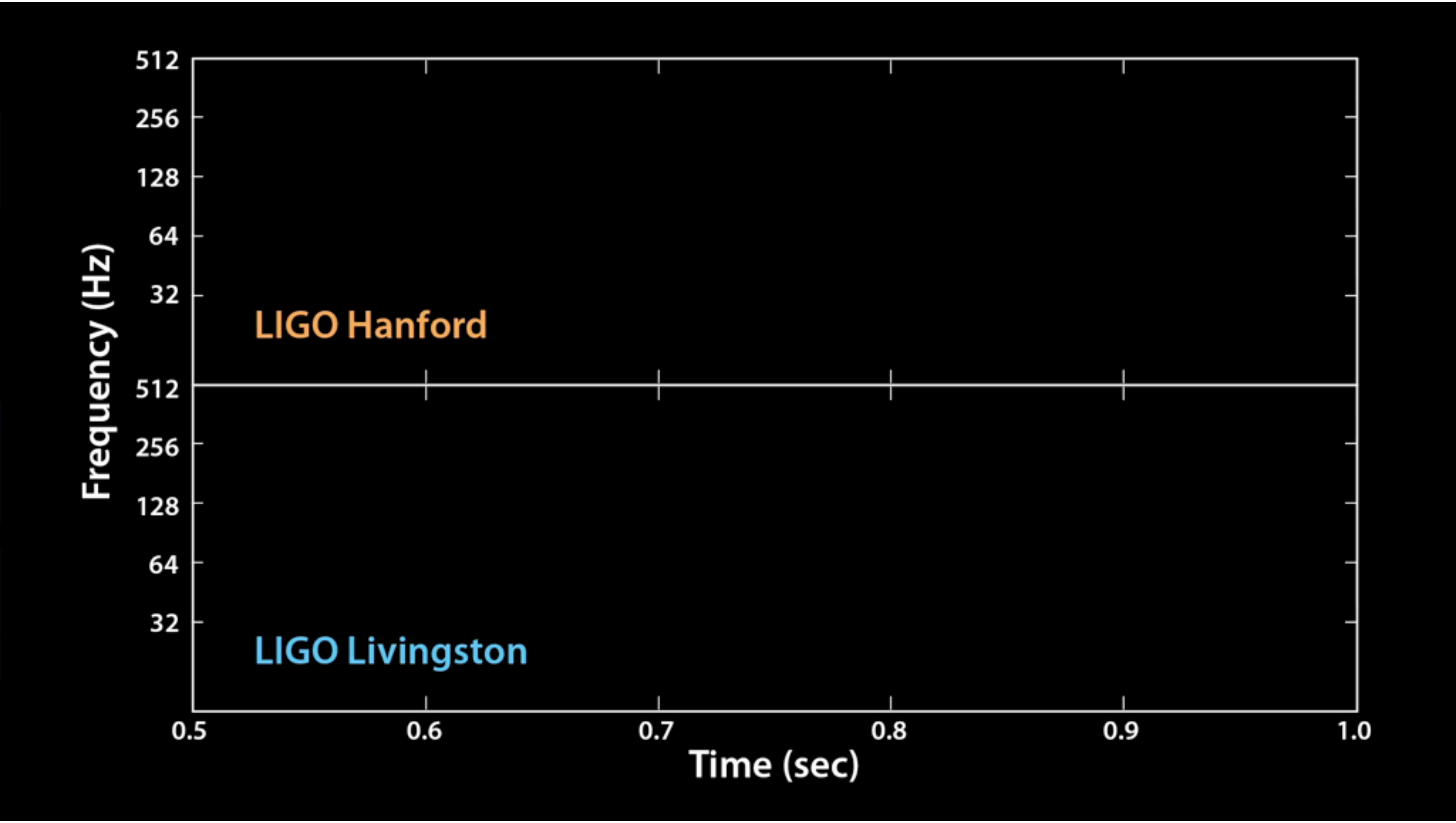
GW150914

VILGCMM16 September 7, 2016

from Abbott et al. 2016, PRL 116, 061102



from Abbott et al. 2016, PRL 116, 061102



www.caltech.edu/gwave

Physical Parameters

- One $36^{+5}_{-4}M_{\odot}$ BH, the other $29^{+4}_{-4}M_{\odot}$. One final BH $62^{+4}_{-4}M_{\odot}$

- Luminosity distance 410^{+180}_{-160} Mpc. Equivalent to a

$z = 0.09^{+0.03}_{-0.04}M_{\odot}$

- $3.0^{+0.5}_{-0.5}M_{\odot}$ radiated away in gravitational waves.

- Final BH spin $0.67^{+0.05}_{-0.07}$

Historic result

- All uncertainties define 90% credible intervals.
- These observations demonstrate the existence of binary stellar-mass black hole systems.
- This is the first direct detection of gravitational waves and,
- the first observation of a binary black hole merger.

Validation I

- Several hours of concurrent operation around event.
- average sensitivity and transient noise typical.
- susceptibility to environmental disturbances checked generating magnetic, radio-frequency, acoustic, and vibration excitations.
- No disturbance recorded by environmental sensors.
- All environmental fluctuations during the relevant second in the instrument are only 6% of the strain amplitude.
- No evidence of disturbances temporally correlated between the two detectors.

Validation II

- 16 days of coincident observations between the two LIGO detectors from September 12 to October 20, 2015 were used for the analysis.

- GW150914 is confidently detected by two different types of searches. One searching for signals from the coalescence of compact objects, using optimal matched filtering with waveforms predicted by general relativity.

- The other search targets a broad range of generic transient signals, with minimal assumptions about waveforms. (burst pipeline).

- Events are assigned a detection-statistic value that ranks their likelihood of being a gravitational-wave signal. The significance of a candidate event is determined by the search background—the rate at which detector noise produces events with a detection-statistic value equal to or higher than the candidate event.

Validation III

- Background is estimated differently for the two searches, but both use a time-shift technique: the time stamps of one detector's data are artificially shifted by an offset that is large compared to the inter-site propagation time, and a new set of events is produced based on this time-shifted data set.

- For instrumental noise that is uncorrelated between detectors this is an effective way to estimate the background.

- If a gravitational wave signal in one detector coincide with time-shifted noise transients in the other, it adds to the background estimate. This leads to an overestimate of the noise background and a more conservative assessment of the significance of candidate events.

Generic transient search:

- excess power in T-F representations.
- The statistics uses a maximum likelihood method where each event is ranked according to

$$\eta_c = \sqrt{2E_c/(1+E_n/E_c)}$$

- where Ec is coherent signal cross correlating 2 reconstructed waveforms, En is the residual noise energy after subtracting the signals. It is a measure of SNR

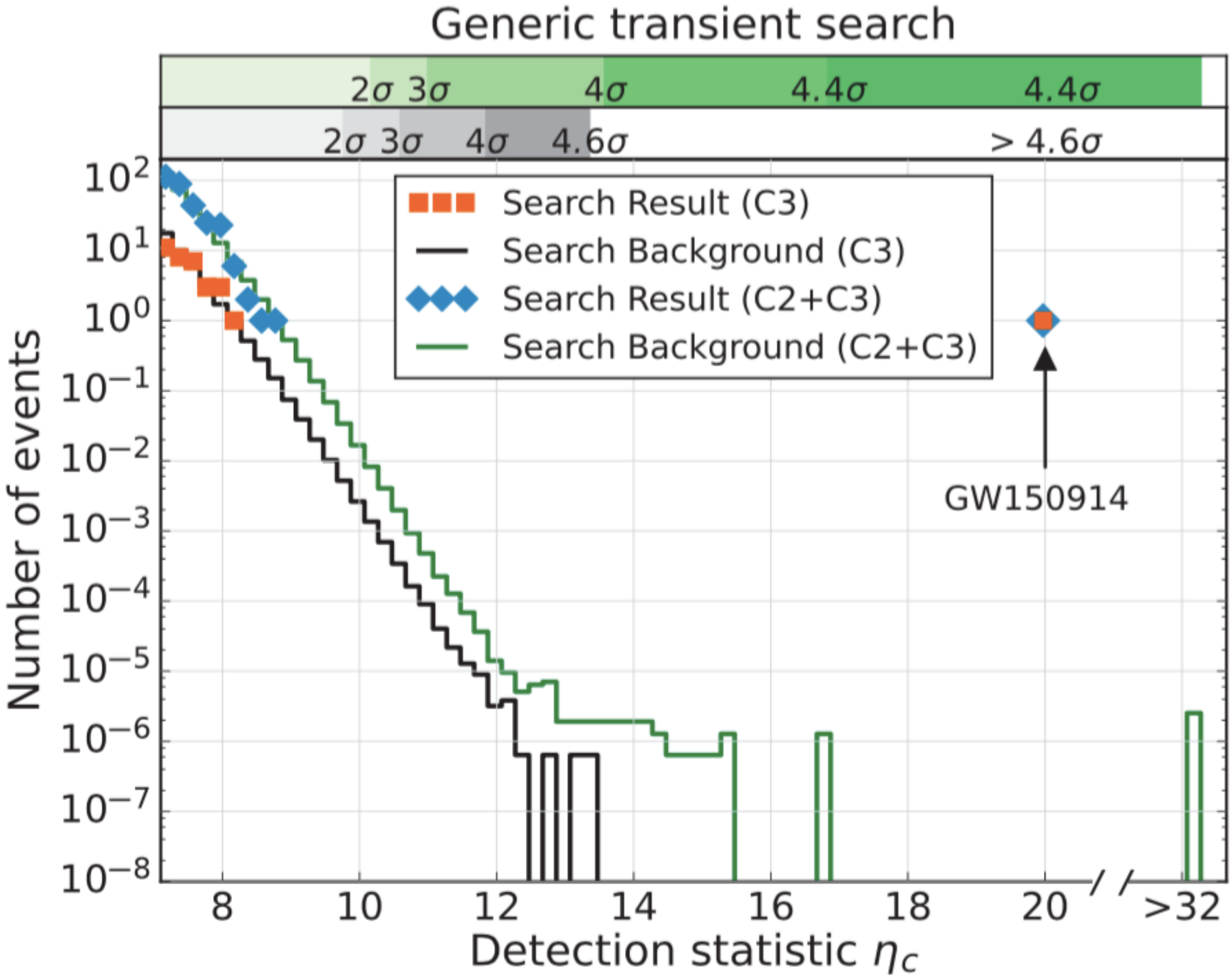
- Based on their time-frequency morphology, three mutually exclusive search classes:

- events with time-frequency morphology of known populations of noise transients (class C1),

- events with frequency that increases with time (class C3),

- and all remaining events (class C2).

- GW150914 is detected with a $\eta_c = 20$ measured on 67400 y. FA lower than 1/22,500 y: 4.6 σ



from Abbott et al. 2016, PRL 116, 061102

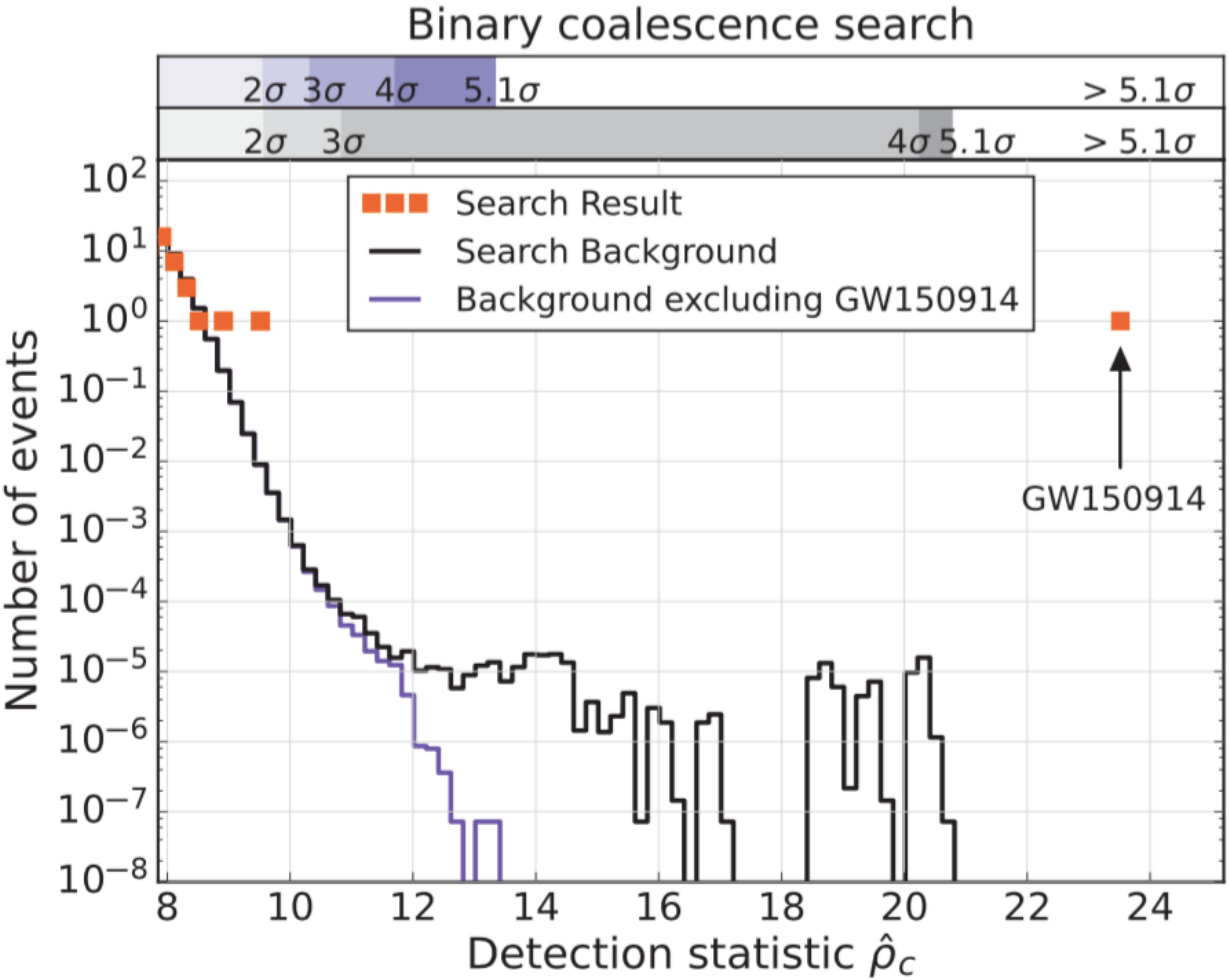
CBC search:

- This search targets gravitational-wave emission from binary systems with individual masses from 1 to 99$M_\odot$ total mass less than 100$M_\odot$, and dimensionless spins up to 0.99.

- 250,000 templates. The search calculates the matched-filter signal-to-noise ratio ρ(t) in each detector. Maxima identified respect to time arrival.

- A χ_r^2 statistics is calculated to test matching of frequencies with template.

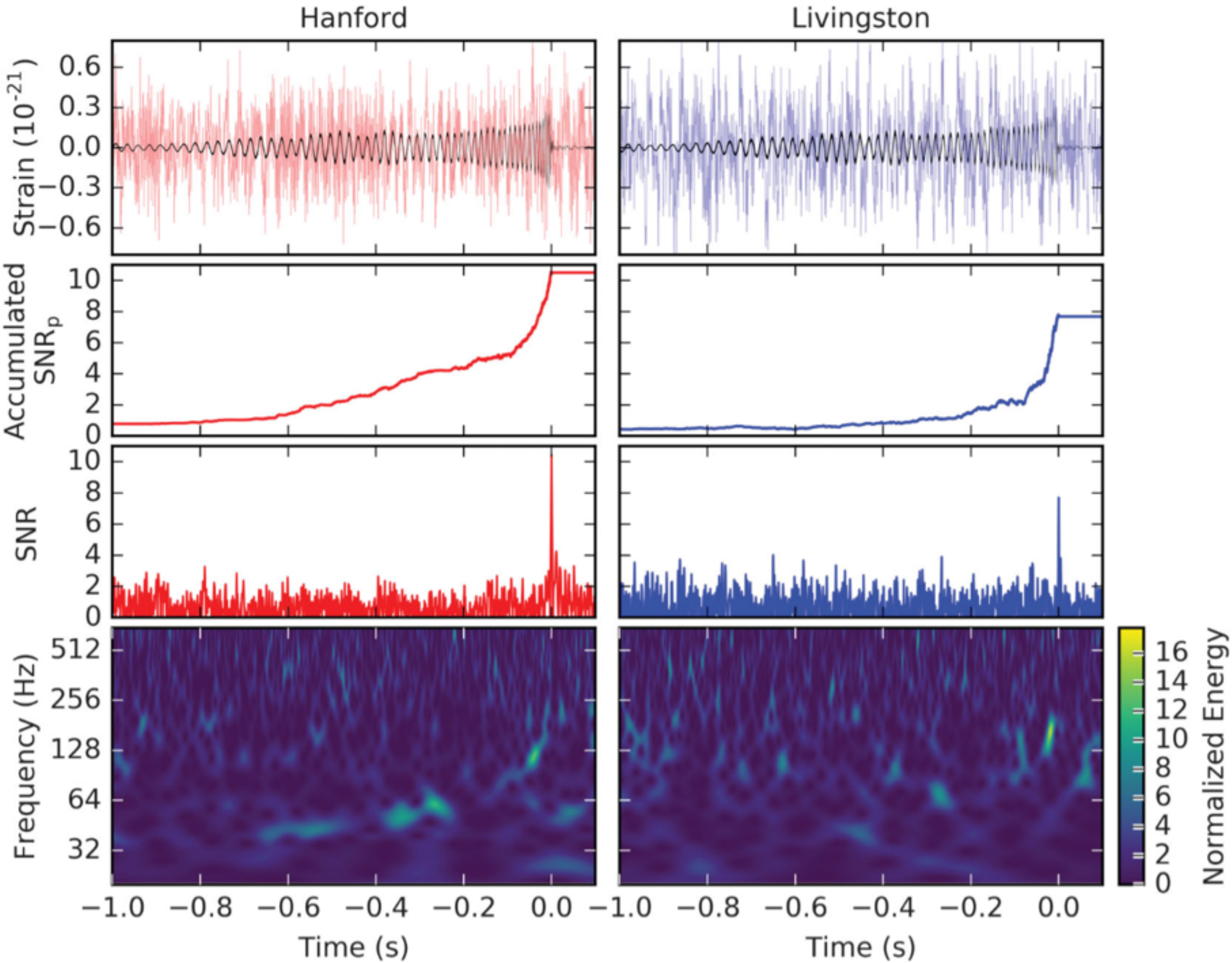
- The final step enforces coincidence between detectors by selecting event pairs that occur within a 15-ms window and come from the same template. Repeated -> 608,000y



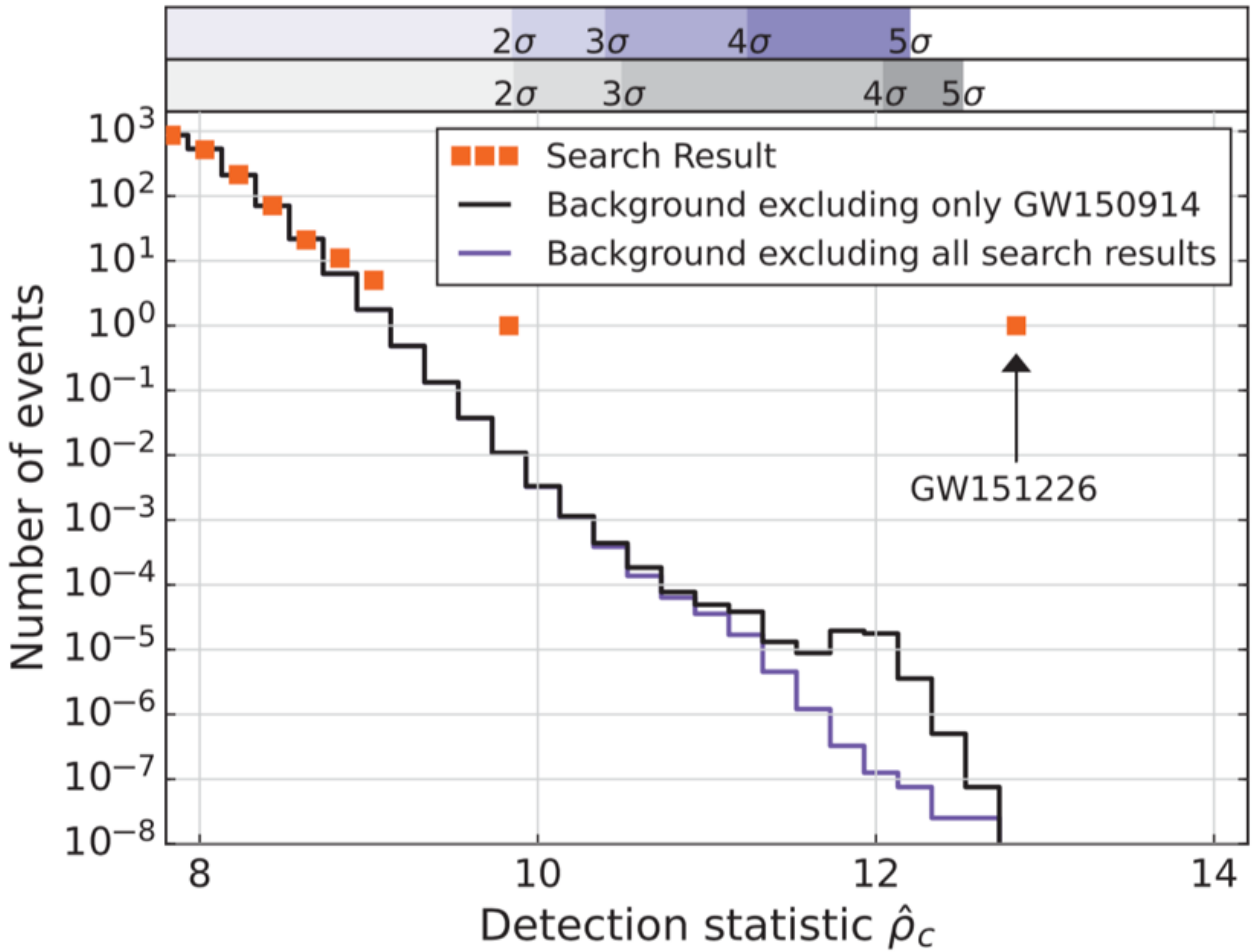
from Abbott et al. 2016, PRL 116, 061102

GW151226

Primary black hole mass	$14.2^{+8.3}_{-3.7} M_{\odot}$
Secondary black hole mass	$7.5^{+2.3}_{-2.3} M_{\odot}$
Chirp mass	$8.9^{+0.3}_{-0.3} M_{\odot}$
Total black hole mass	$21.8^{+5.9}_{-1.7} M_{\odot}$
Final black hole mass	$20.8^{+6.1}_{-1.7} M_{\odot}$
Radiated gravitational-wave energy	$1.0^{+0.1}_{-0.2} M_{\odot} c^2$
Peak luminosity	$3.3^{+0.8}_{-1.6} \times 10^{56}$ erg/s
Final black hole spin	$0.74^{+0.06}_{-0.06}$
Luminosity distance	440^{+180}_{-190} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$



Abbott et al. PRL 116, 241103 (2016



Abbott et al. PRL 116, 241103 (2016

Summary

Event	GW150914	GW151226	LVT151012
Signal-to-noise ratio ρ	23.7	13.0	9.7
False alarm rate $\mathrm{FAR}/\mathrm{yr}^{-1}$	$< 6.0 \times 10^{-7}$	$< 6.0 \times 10^{-7}$	0.37
p-value	7.5×10^{-8}	7.5×10^{-8}	0.045
Significance	$> 5.3\,\sigma$	$> 5.3\,\sigma$	1.7σ